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TEACHER'S MANUAL

TO

WALSH'S NEW GRAMMAR SCHOOL ARITHMETIC

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TEACHER'S MANUAL

TO

NEW GRAMMAR SCHOOL ARITHMETIC.

SPECIAL SUGGESTIONS.

Science of Arithmetic. — In the Manual to the New Primary Arithmetic it is recommended that the teacher lay stress upon the art of computing, leaving the science of arithmetic for a later period. From time to time in the subsequent pages recommendations will be made that the pupils be given more or less acquaintance with the principles underlying the various operations and methods. The teacher should be careful, however, to remember that children are to be taught how to add, subtract, etc., even if they cannot be taught how to express orally or in writing all the reasons for every step taken. The solution of problems that require the pupil to determine the necessary operations will furnish discipline fully as advantageous as that obtained by laying undue stress upon the science of numbers. As the pupils progress from grade to grade, however, the teacher should require more and more technical correctness in their oral and written statements.

Supplementary Work. — It is not expected that the average school will find time for many exercises additional to those provided in the text-book. Some topics may, however, need a larger number of examples than are furnished, time for which may be obtained by the omission of unnecessary

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examples. In order to furnish problems that come within the capacity and experience of the pupils, the teacher should replace some of those in the book by others of his own, the environment of the school and the prevailing industries of the neighborhood supplying the material needed. See Grammar School Manual, Art. 101, page 42.

Omissions. — The teacher may omit such written problems as are too difficult, if his class is backward, as well as those for which he has not the requisite time, commencing first to drop the review problems at the end of each chapter of the arithmetic. For other hints see the General Suggestions preceding the Primary Manual.

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CHAPTER ONE.

WALSH'S NEW GRAMMAR SCHOOL ARITHMETIC, PAGES 1–58.

The major topic for the fifth school year comprises addition, subtraction, multiplication, and division of fractions, with an incidental treatment of reduction of fractions, greatest common divisor, least common multiple, etc. The reviews take up and extend notation and numeration, and include drills in the fundamental processes with simple numbers and United States money; they also include a further development of simple work in denominate numbers and measurements. The new topics introduced are decimals, limited to three places, and bills.

In Chapter One, which is intended for the first half of the fifth school year, the work in fractions is confined to those most frequently used, and whose denominators are small. As the common denominators are obtained by inspection, the introduction of the greatest common divisor and the least common multiple is postponed to the next chapter.

2–3. Pupils are now expected to be more careful to use exact technical language in all their work, especially in fractions, without, however, being required to memorize definitions of the terms they employ.

5–6. Heretofore, the pupils have dealt with but three addends in addition examples. This number is now increased, but in this chapter, as in the last, the common denominator of the fractions is to be determined by inspection. Care should be taken not to slight this chapter in favor of the more difficult work of the next, since the average calculator will seldom be called upon to perform operations with fractions having denominators larger than those here given.

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11. See New Primary Manual, Art. 157.

12. Use these as sight exercises, and require pupils to give the fractional part of the answer in lowest terms.

13. Pupils should not be permitted to write any figures beyond those which appear in the model solution. In this example, the common denominator is 15, the new numerator of the minuend fraction is 9, and that of the subtrahend fraction is 10. This 10 is to be subtracted from a numerator composed of 15 and 9, the two numbers that are just above the 10. The pupils should be required to add 9 and 15, and to take 10 from the sum, without doing any side work. The 7 of 197 should not be changed, except in the mind of the pupils.

If pupils have been previously taught subtraction by the method of increasing the next figure of the subtrahend after "borrowing," they should continue to do so, instead of following the one given in the text-book.

18. In their previous work pupils have used a mixed number in the multiplier but not in the multiplicand.

21. After the pupils have multiplied by a mixed number, their attention should be called to the rule formulated for the multiplication of a whole number by a fraction. They should be shown that in Art. 18, 39 was multiplied by $\frac{3}{4}$ (or $\frac{3}{4}$ was multiplied by 39) by multiplying 39 by 3, and writing the product, 117, over the denominator, 4, giving the result $\frac{117}{4}$, which is reduced to $29\frac{1}{4}$. In the same way the product of 276 and $\frac{3}{8}$ is found by multiplying 276 by 3, and dividing the result by 8; *i.e.* $276 \times \frac{3}{8} = \frac{828}{8} = 103\frac{4}{8} = 103\frac{1}{2}$.

26. If the school year is so short that omissions will be necessary, it might be advisable to limit the pupils to the general method of dividing fractions by the inversion of the divisor. The method here given is used chiefly in oral work, and in written examples in which the fractions have the same denominator.

28. In this grade it is the duty of the teacher to require the pupils to begin to organize and to classify their mathematical

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knowledge. They have discovered in Art. 26, if they did not do so before, that 500 halves divided by 25 halves gives the abstract quotient 20, and at this point No. 2 furnishes a similar illustration.

29. The type of problem No. 2, oral or written, is simple for bright pupils and difficult for dull ones. The average person solves in this way: If $\frac{5}{7}$ of wife's share is \$3500, $\frac{1}{7}$ of her share is one-fifth of \$3500, or \$700, and $\frac{7}{7}$ of her share is 7 times \$700, or \$4900. Many successful teachers prefer to have pupils recognize this problem as one in division. They say that the proper method of indicating the operation needed to solve it is, $\$3500 \div \frac{5}{7}$. Their pupils are required in all problems involving fractions to determine the necessary operation or operations by reading over the problems with integers substituted for the fractions. When this problem is read "5 times wife's share is \$3500," division is clearly indicated, and the pupils, therefore, write "wife's share $= \$3500 \div \frac{5}{7} = \$24{,}500 \div 5$, or \$4900." Teachers who prefer the latter method argue that there is no need to use the intermediate $\frac{1}{7}$ when the value of $\frac{5}{7}$ is given and that of the whole is required. In finding the cost of 3 articles when that of 5 is given, the usual procedure is first to find the cost of 1, by dividing by 5; and these persons claim that when the value of $\frac{5}{7}$ is given, the value of 1 should be found by dividing by $\frac{5}{7}$.

No. 5 must be solved by division, and so must Nos. 7 and 8; it will probably be simpler for the pupils to use the division method to solve Nos. 2 and 3, leaving the method by analysis for problems similar to No. 14, Art. 28, which really involve two operations (unless the ratio method be employed).

Some text-books make a separate topic for such examples as No. 11: "To find what part one number is of another." The first part of this problem is given as a hint to the solution of the second, and pupils should be drilled to use this method of determining the required operation; viz., to read

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the problem omitting the fractional part of a mixed number, or changing a fraction to a whole number.

In No. 12, it is expected that the pupil will employ the relation he found in No. 11 between $3\frac{1}{2}$ and $24\frac{1}{2}$; viz., that the former is $\frac{1}{7}$ of the latter, and that the latter is 7 times the former.

For hints as to work in written problems see General Suggestions and Primary Manual, Art. 157. The teacher will, however, require more accuracy on the part of the pupils in indicating the various steps taken, being careful, also, not to permit them to stretch out their solutions unnecessarily.

33. Pupils will get results more quickly by thinking 6 times 12 hundred, rather than 6 times 1 thousand 2 hundred. The oral answers to all but Nos. 6, 16, and 20, should be given in hundreds.

34. It is not intended that all these examples be worked at one time. The major topic should first receive consideration. Once a week, perhaps, review work should have a place, with, for instance, an example in addition from page 18 and one or two in division from page 20, together with one or two of these multiplication examples.

39. See General Suggestions for hints as to drills, also Art. 160, New Primary Manual.

44. These oral problems are review work, and need not be completed at this stage. They are intended to increase the ability of the pupil to handle large numbers without using the pencil. They should first be solved by the pupils orally from the book, a question being read aloud and the answer promptly given. No analysis should be required, as, with two or three exceptions, there is no likelihood of a mistake in the choice of the operation. At another recitation, the teacher directs the pupils to prepare slips of paper by writing their names, and the numbers from 1 to 10. He directs them to place opposite the first number the answer to, for instance, problem No. 30, giving them a moment to

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read it. At the word "write" pencils are taken, the answer written, and the pencil replaced. The teacher then says, "21," for the next answer, "write," etc., until the answers to ten are written. At a third recitation, ten problems are read by the teacher and the answers written at a signal. When the recitation requires written answers, the teacher sometimes ascertains the number of correct answers to each problem after it is set down; at other times, he waits until all the answers are written. See Art. 82, New Primary Manual, and General Suggestions.

It is expected that at least a few pupils will divide 3 into 18 and 3 quarters directly and get the answer, 6 and 1 quarter, without changing the divisor and the dividend to fourths. The teacher should give one of these an opportunity to explain his discovery to his classmates.

45. By solving these orally with smaller numbers the pupils will generally experience no difficulty in determining the operation. The teacher will find it profitable to employ this method with each set of written problems. He will find, too, that the pupils can obtain the correct answer to many of them without using the pencil at all; the written solution of these latter should not be required.

47. This work affords an opportunity to give some instructions as to the place value of figures.

Pupils should be informed that the use of *and* does not necessarily imply a decimal. Arithmeticians are endeavoring to lead people to omit the use of *and* in reading whole numbers and to employ it only in reading decimals; but they have not as yet been altogether successful.

54. The "unnecessary ciphers" are not to be rejected until after they are counted for "pointing-off" purposes.

57. Some accountants always write the unit figure of the multiplier under the last figure of the multiplicand, so that the decimal (if any) of the multiplier falls to the right of the multiplicand. When the multiplication is completed, the

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decimal point in the product is placed directly under that in the multiplicand. In the case of 35 multiplied by 6.4, the point in the multiplicand is supposed to be after the 5. See Arithmetic, page 85, for further illustrations.

60. Although ciphers may be annexed to the right of the decimal in the dividend, it is just as well to accustom children to dispense with them, and also with the rule for "pointing-off" in the quotient. The pupil in dividing \$12.80 or \$3.60 or \$0.24 by 4, naturally places his decimal point in the quotient when he reaches it in the dividend, and he should be shown the need of doing the same in dividing a decimal by a whole number. The teacher can lead him to see that 12 hundredths divided by 8 gives a quotient of 1 hundredth and something more, which calls for .01 at least, and that when the dividend becomes 120 thousandths the quotient is 15 thousandths, which is .01 with a 5 affixed, and that the proper pointing-off necessarily follows the placing of the decimal point directly under (or over) the decimal point in the dividend, provided he places each quotient figure under (or over) the proper figure in the dividend, as he should be drilled in doing from the time he begins to work examples in short division.

62. These are exercises in the reduction of a common fraction to a decimal.

64. In the preliminary oral solution of these problems, the pupils should be drilled in making a reasonably close approximation to the answer, as a means of testing the written solution. In No. 1, the pupil gives the result as more than 6 yards. In No. 2, he finds the cost of 2 acres to be \$74, and he allows \$20 for the fraction of an acre, it being greater than one half. In No. 3, the pupils may require to be told that "amount" means "sum." In No. 5, the value of the franc may be taken as 20¢, or \$$\frac{1}{5}$. In No. 6, the meter may be taken as 39 inches, which is equivalent to [illegible]e weight of a cubic foot of gold, No. 8, is

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19,400 ounces, or over 1200 pounds. The weight of 128 cu. ft. of water, No. 9, is 128,000 oz., or 8000 lb., or 4 T., which gives .66 times 4 T., or 2.64 T., as the weight of the wood.

While the pupils may find it difficult at first to approximate the answers to some of the problems, they will gradually develop the ability to obtain without the use of the pencil the correct answers to many problems classified as "written."

66–69. To be taken from time to time for review purposes. See Art. 34.

71.

(1) $400¢ \div 10¢$.
(6) $\$7 \div \$\frac{1}{4}$.
(7) $\$20 \div \$\frac{1}{3}$.
(11) $100¢ \div \frac{1}{2}¢$.

74–80. These exercises should be made a feature of the work of this year, and pupils should be required in their written work to employ the short method suggested in these examples. The exercises in 75–78 should be used as drills, the answers being given rapidly.

83–84. These should be treated chiefly as sight exercises, the pupils reading the examples from their books or the blackboard.

85. To prevent the pupils from rewriting the examples, they should each be limited to a small piece of paper large enough to contain only the five or ten answers the teacher may require. The latter calls out the number of the example, and the pupil writes only the result. Pupils have learned before this (Art. 69) that either of the two numbers given in examples similar to these, may be used as the multiplier. Some teachers require this "analysis" to No. 1 when the pupils multiply by 4: "At $1¢$ per lb., the cost would be $687¢$, at $4¢$ per lb., it is 4 times $687¢$, etc." As business men are not given to "analysis," they multiply by 4 anyhow.

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87. In order that pupils may better understand the division exercises in Art. 92, it may be as well to spend a few minutes in showing them that in dividing \$9 by 25¢, which is the regular method of finding the required number of baseballs, they actually only multiply 9 by 4. For, \$9 ÷ 25¢ = \$9 ÷ \$¼ = 36 quarters ÷ 1 quarter = 36 ÷ 1 = 36, contains no real operation except the reduction of \$9 to quarter dollars, the division by 1 amounting to nothing. It can also be shown them that if they analyze the first problem, "At four for \$1, I get 36 for \$9," they are really dividing, only they divide 25¢ into \$1 and multiply the result by 9 as being simpler than to divide it directly into \$9. After this explanation, the pupils should be required to give answers to the remaining examples in this set and to those in Arts. 88–92 as rapidly as possible.

94. These examples should be treated as written problems, the pupils first solving them at sight, making the necessary simplifications. There is no need for any preliminary remarks. The pupils will probably not know that these exercises are difficult unless they are told so. Nothing should be said about reduction ascending or descending, and no particular arrangement of work should be required for the present.

101. The teacher will be careful to add to the examples suggested such others as the conditions will furnish. He will not confine his supplementary problems merely to the calculation of areas, but will draw upon everything available in the environment and interests of his pupils. He will also modify the examples of the text-book to such an extent as to make them conform to the conditions, quantities, prices, etc., of the immediate neighborhood of the school. A book that is intended for use in all parts of this country must be so general in the character and details of its problems as to require many changes in some of the latter in order to make them seem reasonable to the pupils

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in a given locality. While the value of the mathematical training is not affected by the numbers used, children are likely to be distracted by prices and conditions that they consider ridiculous.

103. This bill follows the regular business form. The two vertical lines to the left are used to contain the dates, when the goods are bought at different times (see Arithmetic, page 101). The double space to the right is reserved for the total, which is written on the line just below the last item. The entire cost of each item is written in the next double space to the left. When but one article of a kind is bought, its price is carried directly into the same column. Business men use the dollar mark, as a rule, only in the two places indicated. They do not place a comma after each item, nor do they use the sign "@" before the price.

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CHAPTER TWO.

WALSH'S NEW GRAMMAR SCHOOL ARITHMETIC, PAGES 59–118.

In this chapter, which completes the work usually assigned to the fifth school year, the subject of fractions receives systematic treatment. The other topics contained in the previous chapter obtain additional consideration.

107–115. Pupils are expected to use the terms "factor," "multiple," etc., but they should not be required to memorize the definitions.

116. It is inadvisable to dwell much upon the divisibility of numbers. Practice in reducing, canceling, etc., will accomplish all that is necessary.

If a number divisible by 3 is even, it is divisible by 6. Numbers ending in two ciphers are divisible by 4.

119. Each pupil must determine for himself whether or not to find the greatest common divisor. Time may be lost in looking for common factors. If both terms of a fraction are multiples of 5, as in Nos. 13, 14, 16, 17, 19, 21, etc., or if both are even, as in Nos. 12, 15, 18, 20, and 23, the pupil should commence the reduction by dividing by 2, 4, or 5, according to the conditions; but in the examples from Nos. 4 to 11 many pupils will obtain the answers more quickly by finding the greatest common divisor at once. The "suggestion" is intended for the last twelve examples.

120. The least common multiple, like the greatest common divisor, is a tool to be employed only when it is needed. If the pupils can readily determine the least common denominator by inspection, they should do so.

123. In No. 2, after dividing by 3, the pupils will find it possible to strike out 5, as in the example in Art. 125.

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125. The rules given for addition should not be learned. They are supplied as a brief formulation of the methods employed.

126. Numbers 11 and 12 are introduced to show the relation between common and decimal fractions. See also Nos. 23 and 24 below.

128. Pupils have learned that in multiplying or dividing both terms of a fraction by the same number they do not alter its value; from Nos. 25 and 26 they will see that the value of $\frac{21}{25}$ is changed when both terms are either increased or diminished by 5. In working No. 30, the pupils will first find the product of $4\frac{3}{4}$ and 16, or 16 and $8\frac{7}{8}$, either of which is a whole number, and then complete the work; as they have not as yet had a rule that will enable them to multiply one mixed number by another, they cannot find the continued product of the three in one operation. In No. 33, a parenthesis is introduced to indicate that the product of $3\frac{5}{12}$ and 36 is first to be obtained; this gives a whole number to be multiplied by $8\frac{3}{7}$. The division examples in Nos. 31 and 32 fall within the division method already employed.

129. The word "average" may need explanation in No. 16. No. 18: The second Tuesday is the 8th, and the third Tuesday is the 15th. No. 19: The Thursdays are July 1, 8, 15, 22, 29; Friday is the 30th and Saturday the 31st, Sunday is August 1. No. 20: Some pupils may first take $6\frac{1}{2}$ from $20\frac{1}{2}$, leaving 14, from which $10\frac{3}{4}$ is deducted. No. 21: At 7 pt. for 7¢, or $3\frac{1}{2}$ qt. for 7¢, to find the cost of 12 pt., or 6 qt. In No. 26, the cost of 21 eggs is given, and the cost of 63 eggs — three times as many — is required.

130. In the preliminary sight work, the pupils should not be required to waste time in an attempt to get a close approximation to the answer. Besides being permitted to omit the fractions, they should be advised to change the whole numbers when necessary. The chief reason for the

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oral handling of the problems is to give the slower pupils practice in the employment of a method that will require the careful reading of the conditions of a problem and that will suggest the way to determine the operations needed to solve it. Another reason, but a secondary one at this period, is by estimating beforehand the probable result, to give pupils the habit of avoiding gross inaccuracies in their work. In No. 1, the numbers used might be 48, 57, 40, and 45. In No. 2, the operations will be determined by taking as the numbers either 6 and 19 or 7 and 20. No. 13 should be solved orally by placing on the blackboard the diagram, on which the distances are written in whole numbers, using 40, 40, and 22.

134. In these problems, it will be safer for the average pupil to employ the following method of "analyzing" in order to ascertain the multiplier and the divisor, and then to shorten the work by cancellation.

(1) Cost of 1 horse, $\frac{\$4000}{17}$; of 51, $\frac{\$4000 \times 51}{17}$.

(2) Cost of 1 egg, $\frac{25¢}{15}$; of 10 doz., $\frac{25¢ \times 10 \times 12}{15}$.

(3) One man takes 26 da. $\times$ 18; 13 men, $\frac{26 \text{ da.} \times 18}{13}$.

(4) Number of pounds, 196×17; number of bags, $\frac{196 \times 17}{49}$.

(5) Cost per pound, $\frac{23¢}{7}$; of 91 lb., $\frac{23¢ \times 91}{7}$.

(6) Interest on \$1 for 1 yr., $\frac{\$4}{100}$; on \$650 for 3 yr., $\frac{\$4 \times 650 \times 3}{100}$.

The method of solving by first finding the ratio in lowest terms is of advantage chiefly in a few types of oral problems in which the ratio is a simple one.

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144. Inverting the divisors, we have:

(19) $\frac{20}{1} \times \frac{3}{4} \times \frac{8}{7}$. (20) $\frac{20}{1} \times \frac{8}{7} \times \frac{3}{4}$.

(21) $\frac{20}{1} \times \frac{8}{7} \times \frac{4}{3}$. (22) $\frac{20}{1} \times \frac{8}{7} \times \frac{4}{3}$.

(27) $\frac{17}{3} \times \frac{9}{1} \times \frac{38}{5} \times \frac{5}{23} \times \frac{6}{5}$.

(28) $\frac{11}{2} \times \frac{31}{4} \times \frac{19}{6} \times \frac{25}{4} \times \frac{4}{11} \times \frac{5}{21} \times \frac{1}{31}$.

154. In multiplying and dividing decimals, accountants drop unnecessary figures at the right of the decimal point. The ordinary method of finding the product of 23.1416 by 15.8432, and the accountant's method correct to three decimal places, are given. The accountant shortens his work by excluding all decimals below the fourth place. For this reason,

23.1416	23.1416
15.8432	15.8432
231.416	231.416
115.7080	115.7080
18.51328	18.5133
.925664	.9257
694248	694
462832	46
366.63699712	366.6370
Ans. 366.637—.	*Ans.* 366.637.

when he multiplies by 8, he begins with the third decimal figure of the multiplicand, carrying, however, 5, since the rejected product, 48, is nearly 50. When he multiplies by 4, he begins with the second decimal figure, and so on. In multiplying in this way, the accountant always begins with the left-hand figure of the multiplier. The method of shortening a division example is here shown, which students frequently use in higher work.

```
          23.1416
15/8432)366/6370
      49 7730
       2 2434
         6591
          254
           96
```

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These abbreviated methods of multiplying and dividing should not be taught to elementary pupils.

165. When the answers contain fractions, decimal fractions should be required. Pupils should not, however, be required to use decimals in solving the problems if the use of common fractions shortens the work. See Nos. 7, 8, and 10.

166. While pupils may have been permitted, for practice in decimals, to use the latter, instead of being required to shorten the work by the employment of common fractions, the teacher should now advise the pupils to avoid unnecessary figures. In No. 6, the use of $\frac{3}{4}$ instead of .75 should be recommended.

167. See the preceding chapter of the Arithmetic, Arts. 74–92, also the Manual. Nos. 29, 31, 32, 33, and 35 employ the methods of the other problems.

168. The pupils have already worked similar examples; at this point the matter of arranging the solutions should receive special attention. In No. 2 a departure is suggested from the ordinary form:

37 gal. 3 qt.
4
151 qt.

Many teachers object to this, as the product appears in a denomination different from that of the multiplicand. They also feel that the multiplicand should be 4 qt. and the multiplier 37. The arrangement of the text-book is offered as a compromise. One thing in the solution is important, viz., that the pupil "add in" 3 qt. at the time he performs the multiplication. The amount of circumlocution frequently introduced into arithmetic work prevents pupils from solving as many problems as they should.

169. In Nos. 1 and 2 the pupils must use the same denominate unit, also in 7, 8, 11, 15, and 16. Nos. 3 and 4

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are most easily solved by giving the fractions the same denominator. No. 6: $4\frac{4}{5}$ hr. = 4 hr. 48 min. No. 9: 9 ft. $\div 4 = 2$ ft. 3 in. No. 11: $2\frac{1}{4}$ mo. $\div 12$ mo. $= \frac{9}{4}$ mo. $\div \frac{48}{4}$ mo. $= \frac{9}{48} = \frac{3}{16}$. No. 13: 5 yd. by 6 yd.

170. The correct answers to many of these can be given at sight. In No. 1 the hours foot up 24, or 1 da. No. 2 becomes "What part of 7 da. are $1\frac{3}{4}$, or $\frac{7}{4}$, da.?" No. 3: Interest for 3 years \$180; for $7\frac{1}{2}$ mo. at \$5 per mo., \$36; etc. No. 6: $\frac{3}{4}$ da. No. 7: $.4 \times 24$ gives 9.6 as the number of hours; $\frac{6}{10}$ of an hour = 36 min.

173. See Grammar School Manual, Art. 103, page 43.

174. Use these first as sight exercises, afterwards requiring the pupils to write the answers as the problems are read to them.

175. The pupils should in their written work employ the methods here given for finding products by $12\frac{1}{2}$, 125, 250, etc.

176-177. In these exercises the pupil should be permitted to write first the fractional part of the answer, then the whole number.

179. The pupil writes one figure at a time, but he should not write any unnecessary ones. The answers to the exercises in Arts. 175–179 may be given from the books, the teacher indicating the example by number and furnishing time only for writing the result.

180. (9) More than 7×11. (10) About $\frac{3}{8}$ of 64. As these exercises contain fractions with large denominators, it is inadvisable to solve for the exact answers unless there is plenty of spare time.

181. (3) Nearly 25×12. (4) Nearly $36 \div 6$. (5) More than $86.4 \div .1$. (6) More than 32×6. (7) About $800 \times .1$. (8) Nearly 8×8. (9) More than 7×12. (10) Nearly $64 \div \frac{1}{4}$.

182. (2) 200 yd. at \$2.50. (3) 4 lb. at 25¢. (4) 400 coats at \$12. (5) 8 bu. at \$2. (6) 800 balls at \$$\frac{1}{4}$.

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(8) 1000 lb. at $59\frac{3}{4}¢$. (9) 7 houses at $\$5000$. (10) 507 lb. at $1¢$.

183. In the following, employ the ratio: (5) 10 times as much. (13) Half as long. (49) 3 times as much. (53) 6 times $25¢$. (70) 6 times $5¢$.

184. (7) $\$60800 \times \frac{3}{4} \times \frac{1}{75}$; cancel. (13) $\frac{\$3.08 \times 24}{56}$; cancel. (24) $\frac{\$5.75 \times 15}{12\frac{1}{2}} = \frac{\$5.75 \times 2 \times 15}{25}$; cancel. (35) Make diagram. (43) $1 - (\frac{1}{9} + \frac{3}{8} + \frac{5}{12})$. (50) 4200000 cu. ft. $\div 3\frac{1}{3}$. (51) $\frac{\$80 \times 3}{32}$. (60) $(\$220 \div 2\frac{3}{4}) \times 17\frac{7}{8} = \$220 \times \frac{4}{11} \times \frac{143}{8}$; cancel. (67) $\$36.53 \div 13$.

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CHAPTER THREE.

WALSH'S NEW GRAMMAR SCHOOL ARITHMETIC, PAGES 119–172.

The major topics for the sixth year, the work of which is given in Chapters Three and Four, are decimals and denominate numbers (including measurements). A short, simple treatment of percentage and interest is found in each chapter; it is hoped that the teacher will not carry this too far, using any spare time to improve the other work.

194. Pupils should not be required to annex ciphers for the purpose of having the same number of decimal places in each addend.

199. The method suggested in the last chapter should be followed; viz., that the decimal point in the quotient should be written when it is reached in the dividend.

206. (7) $\$\frac{3}{4} \times 148.6$. (8) $\$.13\frac{1}{4} \times 2376 \div 12 = \frac{\$.53 \times 2376}{4 \times 12}$. (9) $\$35 \times 4.5$. (10) $\frac{\$13.50 \times 28 \times 12}{144}$.

207. See Grammar School Manual, Art. 168.

224. It is inadvisable to dwell too much upon the meaning of percentage. This topic merely introduces a new term, "per cent." The meaning of this expression being given, the pupils should solve the exercises on this page and the next without difficulty.

228. In the preliminary sight work there will be an opportunity to supply any explanations that the pupils may require in order to understand the conditions of the problem.

239. (3) It would support 1 sheep for 21 da. $\times$ 64, and 48 sheep for $\frac{21 \text{ da.} \times 64}{48}$. (4) To build it in 1 da. would re-

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quire 42 men $\times$ 24, to build it in 18 da. requires $\frac{42 \text{ men} \times 24}{18}$.

(11) 6 men take 18 da., 1 man takes 18 da. $\times$ 6, and 9 men (18 boys) take $\frac{18 \text{ da.} \times 6}{9}$.

248. (4) $265\frac{5}{8} \div 43\frac{3}{4}$. (33) See bill Arithmetic, page 50, and Manual, Art. 103. The bill is receipted by writing the following under the items:

Received payment, July 15, 1904,
Thomas Green,
per J. F.

J. F. are the initials of the clerk who signs the receipt on behalf of his employer, Thomas Green.

(43) $\frac{16}{100}$, $\frac{8}{50}$, $\frac{4}{25}$, $.1\frac{8}{8}$.

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CHAPTER FOUR.

WALSH'S NEW GRAMMAR SCHOOL ARITHMETIC, PAGES 173–228.

249. In this chapter the exercises in denominate numbers contain three denominate units, the previous work in this topic being confined, as a rule, to examples limited in each case to two successive denominate units. The teacher should not consider it necessary to require his pupils to work out all the examples, as this topic has but little practical value.

250. See Manual, Art. 168. The arrangement at the right is offered as a substitute for the one given in the arithmetic,

5 gal. 3 qt. 1 pt.
4
20 qt.
3 qt.
23 qt.
2
46 pt.
1 pt.
Ans. 47 pt.

4	2
5 gal. 3 qt.	1 pt.
23 qt.	1 pt.
Ans.	47 pt.

as it dispenses with the cancellation of the 23 qt. after the reduction to pints. The old method is shown at the left.

252. A substitute arrangement is here given together with the objectionable method formerly in use. The 4 and 2 placed at the top may be omitted, their only purpose being to suggest the multiplier or the divisor.

2)67 pt.
4)33 qt. 1 pt.
8 gal. 1 qt. 1 pt.

4	2
	67 pt.
33 qt.	1 pt.
8 gal. 1 qt.	1 pt.

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254. In No. 5 the example really involves the reduction of 63 yd. 3 in. to inches; the 0 ft. is inserted to show the missing denomination. The pupil could solve by multiplying 36 in. by 63, the number of yards, etc., but it is deemed safer to require the average pupil to take but one step at a time, and he is advised to write the missing denominate unit with a cipher prefixed. In No. 6 there is also a missing unit. See Arithmetic, Art. 168, No. 3.

No. 13. $\frac{5}{\cancel{7}}\,\overset{7}{\text{wk.}} = 5\ \overset{24}{\text{da.}} = 120$ hr.

No. 14. $\frac{3}{\cancel{8}\,\cancel{2}}\,\overset{320}{\text{mi.}} = 30\ \overset{5\frac{1}{2}}{\text{rd.}} = 165$ yd.

One way of arranging the work in the solution of No. 13 and 14 is given. The number of units of the next lower denomination, to be used as a multiplier, is written above. Another and a better plan is to write the denomination above and to indicate all the multipliers needed, then to shorten the work by cancellation:

$$(13)\quad \overset{\text{wk.}}{\frac{5}{\cancel{7}}} \times \overset{\text{da.}}{\frac{\cancel{7}}{1}} \times \overset{\text{hr.}}{\frac{24}{1}} = 120 \text{ hr., } \textit{Ans.}$$

$$(14)\quad \overset{\text{mi.}}{\frac{3}{\cancel{32}}} \times \overset{\text{rd.}}{\frac{\overset{5}{\cancel{\overset{10}{320}}}}{1}} \times \overset{\text{yd.}}{\frac{11}{\cancel{2}}} = 165 \text{ yd., } \textit{Ans.}$$

See also Arithmetic, Art. 256, No. 3.

While the teacher must not permit the pupils to waste time in too much unnecessary writing, he should gradually require them to show by their solutions that they are acquainted with arithmetical principles.

257–260. These examples should be taken up only when there is plenty of time.

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264. (1) 18 hr. (2) 31 da. (3) 7 o'clock, 3 o'clock, 5 o'clock. (4) \$18. (5) 25 cases. (6) 21 posts, 2 posts, 3 posts. (7) 31 da., 29 da. (8) 43 da. (9) 23 chapters. (10) 27 problems.

266. (11) 49 da. (12) 60 da. (13) 77 da. (14) 48 da. (15) 40 da. (16) 32 da., etc.

267. In finding the time between dates, some teachers prefer to find first the number of years, then the number of months, then the number of days. In No. 17, there are 13 years from March 3, 1891, to March 3, 1904; 9 months from March 3, 1904, to December 3, 1904; and 29 days from December 3, 1904, to January 1, 1905, December being a month of 31 days. The method given is, however, the one generally followed, and the answers are based upon it.

268. (2) $\frac{1}{4}$ of 16. (6) $\frac{1}{8}$ of 24 hours. (7) $\frac{5}{8}$ of \$12. (9) $\frac{3}{4}$ of 3 gal.

269. Pupils should shorten the work in Nos. 3–8 by employing the fractions $\frac{1}{4}$, $\frac{1}{8}$, $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{6}$, $\frac{1}{16}$. Nearly all of these exercises can be solved at sight.

271. (1) \$3.60 for 1 yr.; $\frac{1}{6}$ of \$3.60 for 2 mo. (2) $\frac{1}{6}$ of \$3.60. (3) \$5 × $2\frac{1}{2}$. (4) $\frac{1}{12}$ of \$6. (5) $\frac{1}{4}$ of \$9. (6) \$4 × $1\frac{1}{4}$. (7) \$3 × 3. (8) \$6 × $2\frac{1}{3}$.

272. In the first example, the 100 in the denominator is canceled by placing a decimal point before 21 in the numerator. In the second example it is canceled by shifting the decimal point in \$160.50 two places to the left.

273.

(1) \$192 × .05 × $3\frac{7}{12}$
(2) \$60 × .04 × $\frac{1}{5}$
(3) \$240 × .06 × $1\frac{1}{12}$
(4) \$14.40 × .05 × $5\frac{5}{12}$
(5) \$36 × .045 × $\frac{77}{360}$
(6) \$99 × .06 × $1\frac{3}{4}$
(7) \$192 × .05 × $2\frac{1}{3}$
(8) \$600 × .04 × $\frac{1}{24}$
(9) \$1200 × .06 × $1\frac{1}{2}$
(10) \$57.60 × .05 × $3\frac{1}{8}$

274. (1) 16 is $\frac{1}{4}$ of 64, or 25 hundredths. (2) 5 is $\frac{1}{5}$ of 25, or 20 %. (3) This becomes: 15 is what part of 16? *Ans.* $\frac{15}{16}$. (4) 16 oz. is what part of 33 oz.? *Ans.* $\frac{16}{33}$.

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(5) $32 \div 3$. (6) 250 pencils. (12) At 1 %, it would be \$3; at $\frac{1}{2}$ %, it is $\frac{1}{2}$ of \$3, or \$1.50. (17) $\$12 \times 4\frac{1}{2}$.

275. (3) $\frac{1}{40}$ of \$5678, or $\$5678 \times .025$. (4) $\$.05\frac{1}{4} \times 5000 \times .03$. (5) $\$720 \times .045 \times \frac{1}{8}\frac{9}{0}$, or $\$720 \times \frac{9}{2}\frac{}{0}\frac{}{0} \times \frac{1}{8}\frac{1}{0}\frac{4}{}$; cancel. (13) 56 pupils $- \frac{1}{7}$ of 56 pupils. (14) $\$45276 + \frac{1}{8}$ of \$45276. (15) He misspelled $6\frac{2}{3}$%, or $\frac{1}{1}\frac{}{5}$, of thirty words. (16) The killed, wounded, etc., amounted to 5% + 12 % + 3 % + 1%, or 21%. Those available for duty constituted 100% − 21%, or 79%. *Ans.* 1000 men × .79.

276. (1) 20 in., 1 ft. 8 in. (2) 12 in., 1 ft. (3) 16 envelopes; 15 sq. in.; 240 sq. in. (4) Number of envelopes, $(50 \times 30) \div (5 \times 3)$, or 100. (5) 24 rugs. (6) 9 sq. in. − 3 sq. in., or 6 sq. in. (7) 40 ft. + 30 ft. + 40 ft. + 30 ft. (8) Number of bunches, 108 sq. ft. ÷ 3 sq. ft., or 36. (9) 12 yd. by 10 yd. *Ans.* 120 sq. yd. (10) 5 yd. + 4 yd. + 5 yd. + 4 yd. (11) 20 in. by 11 in. (12) Number of pieces, $(12 \times 8) \div (2 \times 2)$, or 24.

277. The employment of a diagram will frequently aid the pupil. (13) See table at end of book. (17) Area of lot in square feet: 100×200; of ground covered by house; 25×60.

278. (1) 12 cubes. (2) 12, 24, 36. (3) 12 cu. in., 24 cu. in., 64 cu. in. (4) 4 cubes long, 3 cubes wide, 2 cubes high, is one way. (5) 12 cubes; 12 sq. in. (6) 4 cubes long, 2 cubes wide. (7) The number of cubic inches is equal to product of the length in inches, by the width in inches, by the height in inches. (8) 70 cu. in. (9) 27 one-foot cubes.

281. (3) Volume of the first is 375 cu. ft.; of the second, 376 cu. ft.

282. There are 2150.42 cu. in. in a bushel, 2150.4 is taken, as it lends itself readily to cancellation, being divisible by 2, 3, and 7.

285. See Key. (13) Draw a line 2 inches long. Take a strip of paper 3 inches long and a second strip 4 inches

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long. Place one at each end of the 2-inch line so as to enclose a triangle. Mark the point where the two strips meet, and complete the drawing of the triangle. (14) Draw the two equal sides first. (16) Draw a line of any length, and use two strips of paper equal in length to this line to locate the other sides, as in No. 13. (17) Use a card having a square corner.

288. (5) $40\frac{3}{8} \div 7 = 5$ with a remainder of $5\frac{3}{8}$, or $\frac{43}{8}$; $\frac{1}{7}$ of $\frac{43}{8} = \frac{43}{56}$. *Ans.* $5\frac{43}{56}$.

290. See Grammar School Manual, Art. 45, page 39.

295. The pupils may work these examples from the blackboard, writing only the two partial products and the sum:

(21)	(22)	(23)
195	642	178
$16\frac{1}{4}$	$21\frac{2}{5}$	$7\frac{5}{12}$
$211\frac{1}{4}$	$663\frac{2}{5}$	$185\frac{5}{12}$

296. Use as sight exercises from the blackboard, the pupils writing only the answers.

297–298. It is expected that the pupil will change the decimal to a common fraction when the change will shorten the work.

300. (20) $\$150 \div \frac{2}{5}$, or $\$150 \times \frac{5}{2}$. (21) 2 gal. (24) 8 times $\frac{2}{5}$ yd. (25) $\frac{6}{16\frac{1}{2}}$ rd., or $\frac{12}{88}$ rd. $= \frac{4}{11}$ rd. (28) If six eighths are worth $18\cent$; one eighth is worth $3\cent$. (2) There are 32 qt. in a bushel and I have $10\frac{1}{2}$ qt. etc. (31) 12 times. (36) See Arithmetic, page 50, and Manual, Arts. 103 and 248. (41) See Arithmetic, page 101. (72) $\frac{1}{16}$ of 19712 in.; $\frac{5}{8}$ of 2768 yd.; $\frac{1}{11}$ of 11223344 lb. (73) $\$150 \times .06 \times 2\frac{17}{24}$. (76) Width 42 ft.; perimeter 125 ft. + 42 ft. + 125 ft. + 42 ft.

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CHAPTER FIVE.

WALSH'S NEW GRAMMAR SCHOOL ARITHMETIC, PAGES 229–309.

Each of the previous chapters, from the second of the New Primary Arithmetic to the fourth of the New Grammar School Arithmetic, inclusive, provides for a half-year's work, with its main topics, its reviews, and its preliminary treatment of advanced matter. In Chapter Five, which is intended to cover the arithmetical portions of the work of the seventh school year, the first forty-eight pages are devoted to the main topics, which consist of percentage and its applications, the remaining pages being taken up with reviews and extensions of the work of the earlier chapters, together with a large number of miscellaneous problems, advanced matter being left for Chapter Six.

The first question that confronts the teacher will be the study of the algebraic equations of Chapter Seven. If it is determined that the pupils shall take up this latter chapter in their elementary school course, the work of Chapter Five should be preceded by Arts. 516 to 549 of Chapter Seven, to enable the pupils to apply algebraic methods to the solution of the so-called "problems" in percentage, interest, and bank discount.

302. It is advisable to call to the pupils' attention that they are doing nothing new in these exercises; that they have done the same sort of work in changing common fractions to decimals, except that the decimals in these examples are limited to hundredths, for the remaining places, if any, common fractions being substituted. Those who have worked the percentage problems in Chapters Three and Four may be reminded that the term "per cent" occurs in

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many business transactions: that a real-estate agent, for instance, receives a certain per cent of the amount for which he sells a house or a farm; that a commission merchant who sells produce for a farmer is paid a given per cent of the selling price of the produce; that a savings bank gives its depositors annually, as interest, a certain per cent of the amount on deposit for the requisite time; etc. This preliminary talk should be brief, a fuller treatment being deferred for a few days, to be taken up when Art. 313 is reached.

304. The following need to be emphasized: $\frac{1}{2}\% = \frac{1}{200}$, $\frac{1}{4}\% = \frac{1}{400}$, $\frac{1}{8}\% = \frac{1}{800}$.

305. *Answers:* 300%, 250%, 125%, 420%.

306. SUGGESTIONS:

(5) $\frac{1}{8}$ of 8 pt.
(6) $\frac{3}{8}$ of \$ 24.
(7) $\frac{1}{8}$ of 81 cows.
(8) 6 times $1\frac{1}{2}$ lb.
(9) 4 times $1\frac{1}{4}$ bu.
(10) $\frac{5}{8}$ of 8 qt.
(11) $4\frac{1}{2}$ times \$ 2.
(12) 2 times 99 gal.
(13) $\frac{1}{2}$ of \$ 6.40.
(14) $\frac{1}{4}$ of 8 yd.
(15) $\frac{2}{3}$ of 66 horses.
(16) $\frac{1}{6}$ of 36 in.
(17) 3 times \$ 81.
(18) $\frac{1}{40}$ of 80 sheep.
(19) $\frac{2}{5}$ of \$ 2.50.
(20) $\frac{1}{5}$ of 65 rd.
(21) $\frac{1}{10}$ of 15 lb.
(22) $\frac{1}{80}$ of \$ 60.
(23) $\frac{1}{8}$ of 72 ¢.
(24) $\frac{1}{8}$ of \$ 9.60.

307. It is useless to expect that every pupil will discover the shortest way; and it is inadvisable to require that a pupil should waste the time necessary to work an exercise in deliberating as to the comparative brevity of the different plans that suggest themselves. If, however, a pupil employs during his work on the blackboard a number of unnecessary figures, the teacher should ask if any pupil has used a more direct way, and should permit him to place his work on the board. It may be that several different solutions will be presented, which will naturally lead to a discussion as to

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which is likely to be the best; and the members of the class will gradually appropriate such short cuts as they are able to manage. The preliminary sight work called for in the "Suggestion" on page 232 of the Arithmetic will bring out some of the more obvious means of shortening the work. In the Key will be found the solutions of a few of these exercises.

308. These examples should be used to confirm the statement that percentage contains nothing new. Nos. 1, 2, 4, 5, and 6 can be solved by pupils that never heard of percentage.

The rule does not need consideration if the algebraic method is to be followed in the solution of the exercises.

309. While some educators decry the use of algebraic methods in working problems that can be solved by arithmetic, there are others who do not believe that the pupil necessarily derives the greatest gain in power by being required to do things in the most difficult way. These latter feel that there are sufficient difficulties in the child's school work to make it inadvisable to introduce artificial ones. They claim also that the use of the simplest way of solving problems makes it possible to solve many more in a given time, from which the pupil derives a benefit, while the exclusive use of the difficult method produces discouragement and disgust.

There are two general types of simple problems. The first comprises such as:

Find the sum of 6 and 3.

The minuend is 9, the subtrahend is 6; what is the remainder?

Multiply 8 by 4.

What is the quotient of 32 divided by 8?

These the pupil works out by employing the operation suggested by the words used. In the second type, specimens of which are given below, the required operation is the reverse of the one specified:

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Six added to another number equals 9; find the other number.

The subtrahend is 6, the remainder is 3; find the minuend.

The product of two numbers is 32, one of the numbers is 4; what is the other?

The divisor is 8, the quotient is 4; what is the dividend?

Pupils are aided in the solution of problems of this second type by the arrangement of the numbers in the way indicated by the terms and the operations:

$$6 + ? = 9 \qquad ? - 6 = 3$$
$$4 \times ? = 32 \qquad ? \div 8 = 4$$

When the numbers employed are small, a very young pupil can give at sight the answer to examples arranged in this way, without realizing that he employs subtraction in the first, addition in the second, division in the third, and multiplication in the fourth.

After he has studied algebra, the pupil uses an x instead of the interrogation point, and he employs intelligently the necessary operations. His chief care is to express the equation correctly, and the difficulty of doing this is diminished by the fact that he merely expresses by signs the conditions found in the problem.

One of the difficulties experienced by the careless pupil in solving a "problem" in percentage is in determining which of the two given numbers is the base, another is his neglect to express the rate in hundredths. The average child does not appreciate at a glance that the following are two forms of the same example:

(*a*) 26 is what per cent of 65?

(*b*) What per cent of 65 is 26?

The employment of the equation makes it more difficult for the pupil to blunder, as he does not have any rule to follow. In working the first form of the example (*a*), he writes:

$$26 = \frac{?}{100} \times 65$$

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using the sign of equality for "is," an interrogation point for "what," the denominator 100 for "per cent," a multiplication sign for "of." Substituting an x for the interrogation point, omitting the sign of multiplication, and shifting the terms, he obtains the equation in the regular form:

$$\frac{65\,x}{100} = 26$$

He obtains the same equation from (*b*) which he first writes:

$$\frac{?}{100} \times 65 = 26$$

While it is possible to teach pupils to solve these problems by rule, there is an advantage in making them acquainted with the more general method of the equation. A pupil who has had a reasonable amount of algebraic instruction will be able many years later to reconstruct forgotten arithmetical rules; or, better still, he will be able to continue to solve the problems without learning any rules or feeling the need of any. There is no reason why a seventh-year pupil should be given rules in percentage. When he is asked to find 7 hundredths of 125 he need not be told to multiply 125 by .07. He worked similar examples over three years before, though with simpler fractions, and he has carried on the same kind of work during the succeeding years both in common and in decimal fractions. The same is true of the exercises in this set. They have all been met previously, with the word "fraction" substituted for "per cent." The only excuse is the assumed necessity of working in a short time a sufficiently large number of examples to give children a reasonable familiarity with the new expression "per cent" in its various applications.

In taking up these exercises, it is recommended that the blackboard be first employed, one pupil at a time being sent to work an example, and that the method adopted by the teacher be followed in the solution of each, the object being

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to develop first a general method applicable to all the examples and to postpone the use of shorter methods until the former is mastered.

After all the examples have been worked on the blackboard, the sight solution of the same problems may follow, or the oral exercises of the next page may be taken up, the pupils being now left free to employ their own methods. See Key.

In No. 9, the careless pupil overlooking the "per cent" will consider that 23 is one eighth of the required number, instead of $\frac{1}{800}$. If these examples are solved "at sight," the pupil may be shown in Nos. 10 and 11 that he can omit the common denominator, 5, making the first read "3 is what per cent of 4?" In No. 12 the fractions may be changed to $\frac{15}{20}$ and $\frac{16}{20}$, and the common denominator omitted. In the same way, No. 13 becomes "10 is what per cent of 2?" Some of the pupils who obtain the correct answer to this example, 500%, may fear it is too large and change it to 5%; while others may try $3\frac{1}{8}$ as the base and give 20% as the result. By this time, however, they should have outgrown any habit of guessing at answers and processes they may have indulged in during their earlier school life.

The manner of testing the correctness of the results, applicable to the "algebraic" type of problems, is shown in Nos. 1 and 2. The pupil replaces the "what" in the problems by the answer obtained, as follows:

(1) *40%* of 65 is 26,
(2) 40% of *65* is 26,
(3) 75% of *56* is 42,
(5) *24* is 18% of 150,
(6) 27% of *300* is 81,
(8) 132 is 120% of *110*,
(9) $\frac{1}{8}$% of *18400* is 23,
(10) $\frac{3}{5}$ is *75%* of $\frac{4}{5}$,
(11) $\frac{4}{5}$ is *133⅓%* of $\frac{3}{5}$,
(12) $\frac{3}{4}$ is *93¾%* of $\frac{4}{5}$,
(13) $3\frac{1}{8}$ is *500%* of $\frac{2}{5}$,
etc., etc.,

and he proves each answer by multiplying the base, 65, 56, 150, 300, etc., by the rate expressed as hundredths, .40, .75,

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.18, .27, etc. If the product thus found corresponds with the given percentage, the answer is presumably correct.

It is important that the older pupils be made to understand that the business world requires practically absolute accuracy in all mathematical results. Pupils in the seventh and eighth years should prove all their work. When the proof is obtained by an operation different from that employed in reaching the answer, it may be assumed that the original operations were correctly performed. In an example such as No. 4, which is of the "arithmetical" type, it is not a satisfactory proof to go over again the multiplication of 84 by .15, as the pupil may repeat a mistake made in the first instance. The work should next be done with 84 as the multiplier, or .84 may be multiplied by 5 and this product by 3.

The total of an addition example is proved by adding *down* if the sum is first obtained by adding *up;* the remainder in subtraction is proved by adding the subtrahend to it, to obtain the minuend; the quotient in division is proved by multiplying it by the divisor, and adding the remainder, if any, to the product. The proof in multiplication is suggested above; viz., to use the multiplic and as the multiplier, or to multiply by factors.

The proofs submitted by pupils in their blackboard work should be discussed in order to bring out the essentials of a satisfactory proof.

311. See Key. The suggestion that pupils should solve these examples in their own way is applicable to all kinds of oral work. While the use of these short methods should not be insisted upon in the written work, pupils should not be required to continue to use a roundabout way when they feel able to employ a more direct one.

313–316. It is important to impress upon pupils that commission, insurance, etc., do not constitute new arithmetical topics any more than do cheese, wages, tea, etc. It may

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be shown them that in problems containing these latter terms they are acquainted with the meaning of the other words expressing conditions of price, quantity, etc.; but that unfamiliar words that require explanation will be found in some problems to follow.

In developing the meaning of these new expressions, the teacher should begin with the terms with which his pupils are most likely to be familiar. Some classes may have heard at home more about insurance than about taxes, in which case, insurance should first be discussed. If a policy is procurable, it may be examined and read, at least so far as to show the amount for which the company is liable, and the cost of the insurance. In those sections in which the "valued policy" is in use, the teacher should modify the statement in the text-book that the company pays $ 4000 to the owner of a $ 5000 house that has been damaged to the extent of $ 4000. Some policies provide that the insurer will pay such share of the loss as the amount insured bears to the value; namely, that a $ 4000 insurance on a $ 5000 house will cover only four fifths of the damage done, in which case the company pays only $\frac{4}{5}$ of a $ 4000 loss, or $ 3200. The pupils should be given only such details as have a bearing on the methods in vogue in their immediate neighborhood, and they should not be distracted by the discussion of unnecessary matters. In his treatment of business and other topics the teacher should be careful to ascertain the practices prevailing in the immediate vicinity of the school, and he should make the prices, conditions, etc., conform thereto, regardless of the terms of the problem as found in the text-book.

318. See Art. 157, New Primary Arithmetic Manual, and General Suggestions. The Key furnishes solutions to many of the problems.

319. Profit and Loss is another "application" and not a new topic.

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Answers:

(2) 12½%. (3) $1250. (4) $200. (5) 50%. (6) $16.
(7) 100%. (8) $8. (9) 25%. (10) 25%. (11) 20%.

321. See Art. 103, New Grammar School Arithmetic Manual. Practice in making out bills should be continued. These problems will furnish the necessary data.

324. Many teachers prefer to teach interest by a method of their own, the six per cent method being the one in greatest repute. It is the frequent experience of business men that many pupils taught in this way fail to grasp the underlying principles and find themselves, a few years after leaving school, unable to calculate interest owing to their inability to recall the rule, or remembering it incorrectly, they obtain the wrong result. Pupils should first be drilled in the direct, obvious method of ascertaining results, leaving the short cuts for their later reviews. This does not mean that at any time they should be encouraged or permitted to indulge in roundabout ways or in the use of unnecessary circumlocutions. It does mean, however, that a child asked to find the cost of a given quantity of butter at 19¢ per pound should be expected to employ 19¢ in his earlier calculations rather than 20¢ or 25¢; and that the same pupil, when he regularly takes up the topic of interest, should use the rate given, say 3½%, instead of beginning with 6% and working back to the proper rate. The average child may get the correct result at the time by carefully following some method in which he is drilled; but his attainment is surer if the method is adapted to his immaturity.

In Art. 348 the method of aliquot parts is presented, and in Art. 349 other methods are given, but in each case the pupil employs the rate given in the example, the consideration of the six per cent method being postponed to the following chapter.

The arrangement of the work as shown on page 251 is

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advantageous in all work involving multiplication and division, as it gives the opportunity for shortening the work by cancellation, and it postpones division until all the multiplications are made.

It is not necessary to reduce the time to lowest terms; it is just as well to change the given time to months or days, as the case may be, using the denominator 12 or 360, and making the reduction in the cancellation. See Key.

(6) $\frac{\$720\times7\times21}{100\times360}$ (7) $\frac{\$1000\times5\times8}{100\times360}$

(9) $\frac{\$464.75\times6\times252}{100\times360}$

325. (3) 1080 da. + 180 da. + 9 da.
(5) 720 da. + 5 da.
(6) 450 da. + 27 da.
(7) 360 da. + 150 da. + 29 da.
(8) 2520 da. + 13 da.
(9) $\$1892\times\frac{7}{100}\times\frac{11}{12}$.
(10) 360 da. + 60 da. + 3 da.

328. Nos. 2 and 3 will require no particular explanation. No. 5 resembles No. 4, in the fact that the first payment is not large enough to extinguish the interest then due, and it is considered as not made until September 5, when the sum of the payments, $60, exceeds the interest for six months, $27. The policy of the courts generally is to decline to aid the holder of a note to collect interest on the unpaid interest. In some states, however, laws have been enacted that allow the holder of a note to collect simple interest on the unpaid interest due at the end of the year. The teacher should modify the given rule, if necessary, to conform to the practice of his state. See Arithmetic, page 379.

330. The merchants' rule is interesting rather than valuable.

334. (1) The rule generally favored by persons who do

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not believe that algebraic methods should be brought into elementary arithmetic will be found upon examination to be just as algebraic as the one given lower on the page, without being so simple. By the first method the pupil is directed to calculate the interest on $\$1$ at $4\frac{1}{2}\%$ for 8 months and then to pursue a suggested chain of reasoning; by the second method he naturally calculates the interest on x dollars at $4\frac{1}{2}\%$ for eight months and forms the equation, which practically takes care of itself.

If the algebraic method is employed, there is no need for separate rules to find principal, rate, or time. One rule, which is not a rule, suffices: Represent the required item by x, etc.

Some teachers who dislike the algebraic x employ a French method. They say that the interest, $\$2.88$, represents a principal multiplied by $\frac{9}{200} \times \frac{8}{12}$, which they write out in the form of an equation:

$$P \times \tfrac{9}{200} \times \tfrac{8}{12} = \$2.88.$$

This they solve, not by the algebraic method of clearing of fractions, but by an arithmetical way of making the multipliers of P the divisors of $\$2.88$, thus:

$$P = \$2.88 \times \tfrac{200}{9} \times \tfrac{12}{8},$$

which they solve by cancellation.

336. It is advisable to require children employing the algebraic method to perform the indicated multiplications before making the equation; that is, the latter should not appear in this form in No. 1, Art. 337:

$$2000 \times \frac{x}{100} \times 3 = 300;$$

but should take the one given in the Key:

$$60\,x = 300.$$

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337. The algebraic solutions given in the Key indicate the arithmetical method, it being necessary only to supply the dollar sign.

(1) \$300 ÷ \$60 = rate per cent,
(2) \$144 ÷ \$72 = time in years,
etc.

In the following examples the operations necessary to form the algebraic equations are indicated, the parenthesis being employed to show that the quantities within it should be combined before the equation is written:

(7) $\left(460 \times \frac{7}{200} \times x\right) = (485.15 - 460)$

(9) $x + \left(x \times \frac{3}{100} \times 8\right) = 6200$

(10) $\left(7548 \times \frac{x}{100} \times \frac{95}{360}\right) = 119.51$

(11) $\left(9000 \times \frac{4}{100} \times x\right) = 632$

(12) $x + \left(x \times \frac{5}{100} \times \frac{830}{360}\right) = 160.60$

(14) $\left(120 \times \frac{x}{100} \times \frac{435}{360}\right) = 4.35$

(15) $\left[(97.57 - 7.57) \times \frac{4}{100} \times x\right] = 7.57$

(16) $x + \left(x \times \frac{9}{200} \times \frac{1339}{360}\right) = 93.39$

(18) $\left(x \times \frac{5}{100} \times \frac{1643}{360}\right) = 16.43$

Pupils should be required to prove each example. See Art. 309, page 64.

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338. See Key.

340. See Key. In the examples in bank discount the interest is taken for the exact number of days, on the basis of a year of 360 days. Some banks in discounting the given note take interest for one-fourth of a year regardless of the number of days in the months; other banks take the interest for $\frac{92}{365}$ of a year. In states in which days of grace are allowed, interest is taken for three additional days.

Answers are given in the Appendix at the end of this chapter, allowing 3 days of grace, and following the method of ascertaining the exact number of days between the date of discount and the date of maturity, the year being considered as one of 360 days.

347. Pupils should be required to prove their answers.

348. Many accountants use the method by aliquot parts in many of their calculations. They would probably employ it in finding interest were it not that they use interest tables for this purpose.

356. (12) Algebraic method:

Let x represent B's share.

Then $x + 15 =$ A's share

$x + 27 =$ C's share

$$x + x + 15 + x + 27 = 320.$$

360. See Manual, Art. 250.

362. See Manual, Art. 252.

369. (1) 9 times 4 yd. = 36 yd. Dividing 36 yd. by $5\frac{1}{2}$ yd. we get a quotient of 6 (the number of yards) with a remainder of 3 yd. See Key.

374. (2) In 1 rd. there are $16\frac{1}{2}$ ft.; in 2 rd. there are 33 ft.; in $\frac{1}{4}$ rd. there are $\frac{1}{4}$ of $16\frac{1}{2}$ ft., or $4\frac{1}{8}$ ft.

Ans. 33 ft. + $4\frac{1}{8}$ ft., or $37\frac{1}{8}$ ft.

(3) 4 bu. = 16 pk., worth at \$ $\frac{3}{8}$ per peck, \$6.

(4) $97\frac{1}{2}¢ \times 20$ or \$9.75 by 2.

(5) \$4 = 4000 mills. *Ans.* 2000 pencils. Or, 5 pencils for 1¢, 500 for \$1, 2000 for \$4.

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(6) 2000 lb. for 500¢ = 4 lb. for 1¢.

(7) At 4 lb. for 1¢, 480 lb. cost $1.20. 3 T. cost $15. 3 T. 480 lb. cost $15 + $1.20, or $16.20, *Ans.*

(8) For $10, 2 T. can be bought; for 80¢, at 4 lb. for 1¢, 320 lb. *Ans.*, 2 T. 320 lb.

(9) 2 yd. 2 ft. = 8 ft.; 4 yd. 1 ft. = 13 ft. *Ans.* 65¢.

(13) $\frac{5}{6}$ lb. Troy = 10 oz. *Ans.* $1.20.

(14) $\frac{1}{4}$ mi. = 80 rd. = 80 times $16\frac{1}{2}$ ft., or 1320 ft., *Ans.*

(15) 2 lb. 10 oz. = 34 oz. *Ans.* 17 tablespoons.

375. See Key. (5) The work will take this form before canceling: $\frac{825 \times 1000 \times 9}{2 \times 10 \times 24 \times 20}$ = number of ounces.

(9) 109 qt. is what per cent of 654 qt.?

(10) $\frac{47520 \times 20 \times 2}{320 \times 11 \times 3 \times 12}$ = number of miles.

377. (10) If it requires 60 yd. of carpet 6 eighths wide it will require $\frac{6}{8}$ of 60 yd., when it is 5 eighths wide.

378. (5) The interest on $350 for $3\frac{1}{2}$ years at 4%.

(6) 32 horses will consume $\frac{32}{48}$, or $\frac{2}{3}$ of 180 bu. in 10 da. *Ans.*, 120 bu. 12 bu. per day = 144 bu. in 12 da. and 180 bu. in 15 da.

380. (11) $\frac{\$128 \times 520 \times 36}{160 \times 48}$.

(12) $\frac{8 \text{ da.} \times 12 \times 8 \times 84}{8 \times 10 \times 96} = 8\frac{4}{10}$ da. $\frac{4}{10}$ of a 10-hour day = 4 hr. *Ans.* 8 da. 4 hr.

(14) $\frac{54 \text{ rd.} \times 32 \times 9}{27 \times 6}$. (15) $\frac{90 \text{ da.} \times 50 \times 8}{72 \times 10}$.

(17) $\frac{68 \text{ men} \times 32 \times 15}{34 \times 8}$. (18) $\frac{500 \text{ men} \times 48 \times 27}{60 \times 30}$.

(19) $\frac{8 \text{ hr.} \times 12 \times 15}{9 \times 16}$.

(24) Both do $\frac{1}{24} + \frac{1}{48}$, or $\frac{1}{16}$, of the work in 1 day.

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381. See Key.

383. The pupils as they go forward in the grades lose their ability to handle numbers readily and accurately unless practice is kept up. These shorter ways of obtaining products will probably make the work more interesting.

385. The pupils should use no more figures than are employed in the model solutions.

386. (2) Multiply the denominators of $\frac{5}{7}$ and $\frac{25}{46}$; divide the numerators of $\frac{12}{18}$ and $\frac{84}{108}$.

(13) Divide the denominators of the last two.

(14) $30\frac{2}{7} \times \frac{8}{8} \times \frac{1}{7}$. (23) 320 rd. $\times \frac{4}{15}$. (25) $\frac{3}{4} \times \frac{41}{8} \times \frac{36}{5} \times \frac{5}{9}$.

387. (3) See Arithmetic, Art. 199.

(4, 8) See Arithmetic, Art. 162.

(10) $6s.\ 9d. = 6.75s.$ To change to pounds, divide 6.75 by 20, or .675 by 2.

(17) Change the divisor (denominator) to a whole number by omitting the decimal point, making five fewer decimal places in the dividend (numerator): $\frac{35 \times .56}{7}$, or $\frac{.35 \times 56}{7}$.

(18) Change $\frac{69}{320}$ to a decimal; that is, divide 6.9 by 32.

(19) .4285 mo. = 30 da. × .4285, or 12.855 da.

.855 da. = 24 hr. × .855, or 20.52 hr.

.52 hr. = 60 min. × .52, or 31.2 min.

.2 min. = 60 sec. × .2, or 12 sec.

Ans. 12 da. 20 hr. 31 min. 12 sec.

388. See New Grammar School Arithmetic, page x.

(1) In doing the work aloud the pupil says ten fifty-six, eleven six, eleven twelve, omitting the word "hundred."

(6, 7) Omit the word "dollars" also in the preliminary drill, using the mark, however, in the written answer.

(30–33) These are especially valuable. The pupil says 27 hundred less 27, twenty-six seventy-three; 36 hundred less 36, thirty-five sixty-four; etc.

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389. These problems can all be worked without a pencil by the average pupil. They should first be solved from the book (see New Grammar School Manual, Art. 44).

(4) Three forty-five + one fifty = four ninety-five. Adding a dollar and deducting a cent gives five ninety-four.

(5) $25\frac{1}{4} - 8(17\frac{1}{4}) - \frac{1}{2} = 16\frac{3}{4}$.

(6) 5¢ per quart, 40¢ per peck, $1.60 per bushel, $3.20 for 2 bu.

(7) $83\frac{1}{4} \div 9 = 9$, with a remainder of $2\frac{1}{4}$, or $\frac{9}{4}$. $\frac{1}{9}$ of 9 fourths = $\frac{1}{4}$. *Ans.* $9\frac{1}{4}$ yd.

(8) 10 − 6 (4) − .30 (3.70) − .07 = 3.63. The pupil thinks ten less six (4) less 30¢ (3.70) less 7¢.

(9) Cost at 5¢, $1.20; deduct 6¢ ($\frac{1}{4}$¢ × 24). *Ans.* $1.14.

(10) $19 + $\frac{1}{4}$ of $19.

(11) At $2 per yard, $240; at $\frac{1}{8}$, $15.

Ans. $240 − $15, or $225.

(12) $120 \times \frac{8}{15}$. (20) Use 28 as the multiplier.

(23) At $\frac{1}{2}$ each, $8. *Ans.* $8 − 16¢.

(24) 27 hundred × $\frac{3}{4}$ = 81 hundred ÷ 4 = 2025.

(26) 9 hundred ÷ $\frac{3}{4}$ hundred = 9 × $\frac{4}{3}$.

(32) Use 42 as the multiplier. 1260 + 126.

(33) 4960 ÷ 160 = 496 ÷ 16. (34) $320 \times 5\frac{1}{2} \times 3$.

(37) (80¢ + $1.50) ÷ (4 + 6). (38) $99 \div \frac{11}{4}$, or $\frac{4}{11}$ of 99.

(45) 160 men do it in 1 da., $\frac{1}{5}$ of 160 men in 5 da.

390. (7) 800.0046 × .003040.

(14) ($25 × .096) + ($3 × 1.65) = ($\frac{1}{4}$ of $9.60) + ($1.65 × 3).

(17) 100 per minute = 6000 per hour. Number of hours, 1000000 ÷ 600 = $166\frac{2}{3}$. At 8 hr. per day, $166\frac{2}{3}$ hr. = 20 da. $6\frac{2}{3}$ hr., or 20 da. 6 hr. 40 min.

(18) 29 da. + 30 da. + 31 da. + 30 da. + 31 da. + 11 da.

(19) Time, 130 da. (leap year). (23) $900 + $\frac{1}{8}$ of $900.

(25) .25, 25%; .125, $12\frac{1}{2}$%; $\frac{3}{8}$, $37\frac{1}{2}$%; $1\frac{1}{4}$, 1.25, 125%; $\frac{1}{6}$, $16\frac{2}{3}$%; etc.

(26) Time, 1 yr. 11 mo. 2 da.

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(30) Number of quarts: 202500 + 6, or 33750. Average quantity per cow: 33,750 qt. + 25, or 1350 qt.

(50) Total cost: \$480 + \$30, or \$510. To gain 30% of \$510, or \$153, I must sell 80 bbl. for \$510 + \$153, or \$663. Selling price per barrel, \$663 ÷ 80, or \$8.28¾, *Ans.*

(53) $\frac{\$1030 \times 4.75}{12.875}$, or $\frac{\$1030 \times 4750}{12875}$; cancel.

(55) 3 fifths of a ton will cost ⅓ as much as 9 fifths, or ⅓ of \$18¾, or \$6¼, *Ans.*

(56) $\frac{1584 \text{ yd.}}{11 \times 9}$, *Ans.* $\frac{1584 \text{ yd.} \times 6 \times 7}{11 \times 9}$, *Ans.*

(58) $11\frac{3}{8} - 8\frac{3}{4} = 11\frac{3}{12} - 8\frac{9}{12} = 2\frac{11}{12}$. $8\frac{3}{4} \times 11\frac{3}{8} \div 2\frac{11}{12} = \frac{35}{4} \times \frac{35}{8} \times \frac{12}{35}$, cancel.

(61) Number of dollars: $\frac{7000 \times 165}{412\frac{1}{2}} = \frac{7000 \times 165 \times 2}{825}$.

(64) $\frac{2}{5} \times \frac{5}{8} \times \frac{15}{4} = \frac{25}{12} = 2\frac{1}{12}$. $2\frac{1}{12} + 8\frac{2}{3} = 10\frac{3}{4}$. $10\frac{1}{2} - 7\frac{11}{12} = 2\frac{7}{12}$. $10\frac{3}{4} \div 2\frac{7}{12} = \frac{43}{4} \times \frac{12}{31} = \frac{129}{31} = 4\frac{5}{31}$, *Ans.*

(65) Number of bushels: $\frac{102 \times 51 \times 45}{2150.4} = \frac{1020 \times 51 \times 45}{21504}$; cancel.

(82) G. C. D. 40. L. C. M. 60.

(90) Find L. C. M. of 48, 20, 21 (omitting 24). See Arithmetic, Art. 122.

(102) ⅘ of \$210 + ⅝ of \$154.

(104) Interest for 2½ yr. = \$593.75 − \$500, or \$93.75; for 1 yr. = \$93.75 ÷ 2½, or \$37.50. For \$500 to produce \$37.50 per year, the rate per cent must be 37½ ÷ 5, or 7½.

(115) Cost per book, \$54 ÷ 12, or \$4.50; profit, ⅕ of \$4.50, or \$.90; selling price, \$4.50 + \$.90, or \$5.40.

(127) $\frac{3000000 \times .024}{6000} = \frac{3 \times 24}{6}$.

(128) \$2880 ÷ $\frac{1}{20}$, or \$2880 × 20.

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APPENDIX TO CHAPTER FIVE.

Days of Grace

The following answers include three days of grace in each case, regardless of the fact that the note may be drawn in a state in which grace is not allowed. It is expected that pupils in states allowing grace will use these days in computing discount. Sundays and holidays are not taken into consideration.

340. This note matures 3 da. after Aug. 16, *i. e.* on Aug. 19, 95 da. after date.

Face of note	$300.00
Discount 95 da.	4.75
Proceeds	$295.25

341. (1) $ .41+
(2) $ .55+
(3) $2.89+
(4) $ .37−
(5) $6.89−
(6) $173.59½
(7) $ 36.23+
(8) $791.42+
(9) $176.74½
(10) $985.41⅔

342. (11) Due March 9, 32 da. $3.20
(12) $2.60
(13) $5.64
(14) $1.74−
(15) $5.38−
(16) $2.52
(17) $5.58

343.

	Matures	Term	Discount	Proceeds
(18)	Aug. 18, 1902	7 da.	$ .10+	$ 87.50−
(19)	Jan. 12, 1903	55 da.	$1.52−	$122.66+
(20)	Jan. 7, 1903	26 da.	$2.55+	$502.05−
(21)	Feb. 17, 1904	59 da.	$ .71+	$ 71.65−
(22)	Jan. 31, 1904	26 da.	$1.01+	$232.99−
(23)	March 5, 1904	21 da.	$ .34−	$ 95.56+
(24)	Oct. 7, 1904	37 da.	$1.35−	$162.65+
(25)	June 9, 1904	92 da.	$1.28−	$ 81.92+

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344.

	Amount	Term	Discount	Proceeds
(1)	$150.06–	17 da.	$ .43–	$149.63+
(2)	$177.71+	60 da.	$ 1.78–	$175.94–
(3)	$353.68–	53 da.	$ 3.12+	$350.55+
(4)	$855.19	47 da.	$ 8.93+	$846.26–
(5)	$735.	71 da.	$10.15–	$724.85+
(6)	$510.42–	44 da.	$ 3.74+	$506.68–

347. (1) $106.33–
(2) $6\frac{24}{31}\%$
(5) $4.68
(8) $1200
(9) $304.26–
(11) $853.27+
(12) $56.62½

349. (16)

Face . . .	$873.60
60 da.	8.736 } Deduct
30 da.	4.368 } Deduct
3 da.	.4368 } Deduct
	$860.0592
Proceeds . .	$860.06–

(17) $24.74–
(18) $761.06–

350. (26) $252.37+
(27) $13.09–
(28) $486.10–
(29) $2.33+

390. Page 304. (69) $594.50
Page 305. (77) $77.95

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CHAPTER SIX.

WALSH'S NEW GRAMMAR SCHOOL ARITHMETIC, PAGES 310–414.

391. The practice in this country of giving to the colon (:) a different arithmetical meaning from that given to the sign of division (÷) makes a simple and familiar topic seem like a new one. Upon the European continent the colon (:) is the sign of division and our mark (÷) is unknown, so that a pupil understands that 3:6 means $\frac{3}{6}$, or $\frac{1}{2}$, which he also writes 1:2 without any special instruction. Many teachers, who have been unable to obtain the wonderful results in arithmetic that were said to follow the use of the "ratio method," are in favor of dropping ratio and proportion entirely from the elementary text-books. While it is possible to solve all ordinary arithmetical problems without employing proportion, so-called, there is an advantage resulting from the comparison of numbers and quantities that makes it advisable to take up at least the oral exercises and problems. Comparing and measuring should be done all through the course in arithmetic, but there is no need of pursuing these lines to the extent advised by the believers in the "ratio method."

When children are ready for problems involving two operations, one like the following is given: If 2 oranges cost 6 cents, what will be the cost of 5 oranges? This the pupil solves by ascertaining the cost of 1 orange, and multiplying the result by 5, the operation being $(6¢ \div 2) \times 5$. Somewhat later this problem may be given: Find the cost of 6 oranges at the rate of 3 oranges for 5¢. Without ever having heard of ratio, he naturally thinks of the cost of 6 oranges as being double that of 3 oranges. The operations

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are $5¢ \times (6 \div 3)$. Both types of problems should frequently be given to be solved orally, with perhaps an occasional hint to the unobservant pupil to notice the relation between the quantities in a problem of the second type. In the written work, however, there is no particular need of distinguishing between these types. If the pupils are properly taught, they will develop the habit of indicating the operations before commencing to calculate, and cancellation will make the work the same for each type. The "written" problems supplied in some books to show the advantages of the "ratio method" bear the evidences of manufacture, and are not such as occur with sufficient frequency in real life to make it advisable to give them special consideration in school.

394. (3) 260 min. ÷ 195 min.; cancel.
(4) $5\frac{3}{16}¢ \div 25\frac{1}{8}¢$.
(6) $(600 \div 8\frac{1}{7}) \div (300 \div 3\frac{1}{3}) = 70 \div 90$; reduce.

395. (1) $16\frac{1}{2}$ ft. to $5\frac{1}{2}$ ft. 3 to 1, *Ans.*
(2) 7 hr. to 24 hr. 7 to 24, *Ans.*
(3) $30¢$ to $6¢$. 5 to 1, *Ans.*
(4) A's speed is greater. 5 to 4, *Ans.*
(5) E's daily earnings are larger. 4 to 3, *Ans.*
(6) 3 to 4.
(7) 7 to 22.
(8) 8 to 9.

396. The pupils should be permitted to attempt to work these without assistance. No. 1 requires that $\frac{3}{16}$ be changed to a fraction having 64 for a denominator. No. 2 is a variation of No 1. In No. 4, the 3 bu. are changed to 12 pk., making the first couplet $\frac{1 \text{ pk.}}{12 \text{ pk.}}$, which requires \$2 in the numerator of the second couplet. The missing items are: (5) 40¢; (6) 63; (7) 35; (8) 19; (9) \$48; (10) 20; (11) 9 horses; (12) 2 yd.; (13) 12¢; (14) $10(\frac{10}{8})$; (15) .02.

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399. (9) $\frac{1}{4}x = \frac{2}{3} \times \frac{3}{5}$; $x = \frac{2}{3} \times \frac{3}{5} \div \frac{1}{4} = \frac{2}{3} \times \frac{3}{5} \times \frac{4}{1}$.

(10) $\frac{3}{5}x = 2\frac{1}{4} \times \frac{1}{3}$; $x = 2\frac{1}{4} \times \frac{1}{3} \div \frac{3}{5} = \frac{9}{4} \times \frac{1}{3} \times \frac{5}{3}$.

(11) $x = \frac{5}{8} \times 1\frac{3}{5}$.

(12) $3\frac{1}{8}x = \frac{5}{8} \times 11$; $x = \frac{5}{8} \times 11 \div 3\frac{1}{8} = \frac{5}{8} \times \frac{11}{1} \times \frac{8}{25}$.

(13) $x \times x = 36$. As the pupils have not had square root, they may not see the answer at once; but they should not be told it.

(16) $6x = 15 \times 4$; $x = 15 \times 4 \div 6 = 10$. *Ans.* 10 bottles.

(17) $36x = 16 \times 9$; $x = 16 \times 9 \div 36 = 4$. *Ans.* 4 men.

400. (2) 23¢ × 7. (3) 12 hats × 4. (4) 75 revolutions × 12. (5) \$6 × 4½. (6) 75¢ × 8. (7) 3 mi. × 15. (8) 10 mo. ÷ 5. (9) 15 da. ÷ 3. (10) 270¢ ÷ 3.

401. Pupils should not be required to work these problems in any one way. Some will multiply \$267 by 4 at once, which is the best method. Another way is to indicate the cost of one, $\frac{\$\,267}{9}$, and then the cost of 36, $\frac{\$\,267}{9} \times 36$, to be worked by cancellation. This is the most general plan and requires the same operation as the first, but with the writing of a larger number of figures. A third method is given in the text-book. The method that should be forbidden is the one in which the pupil first finds the cost of 1 cow. See Key.

(5) $\frac{42 \text{ yd.} \times 165}{9} \cdot \frac{9 \text{ lb.} \times 196}{42}$.

(6) $\frac{\$\,357 \times 24}{17}$. (7) $\frac{27 \text{ da.} \times 17}{51}$.

404. (5) $1000 : 900 :: 50 \text{ lb.} : x \text{ lb.} = 45 \text{ lb.}$

$1000 : 90 :: 50 \text{ lb.} : y \text{ lb.} = 4\frac{1}{2} \text{ lb.}$

$1000 : 10 :: 50 \text{ lb.} : z \text{ lb.} = \frac{1}{2} \text{ lb.}$

Cancellation should be resorted to when possible.

(6) $27 : 15 :: \$\,45 : \$\,x$, $27 : 12 :: \$\,45 : \$\,y$.

(7) $50 : 27 :: \$\,30 : \$\,x$, $50 : 23 :: \$\,30 : \$\,y$.

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Pupils that solve this problem by finding 60¢ the cost of the boat for a day, etc., should be encouraged to apply their own methods to the other problems.

405. The note at the bottom of page 321 of the Arithmetic should be discussed. The pupils should be informed that partners generally draw up a written contract, which states the amount contributed by each, the length of time the partnership is to continue, the manner of dividing the profits, etc. It is seldom that such a partnership is formed as that described in Nos. 4, 9, 10, or 11. When two men contribute different sums to the partnership, it is frequently agreed that the one furnishing the larger amount shall receive, out of the profits, interest on the difference between the two shares, the remainder of the profits then to be divided equally.

(2) 2 36
2020 : 720 : : $101 : A's share = $36.
2 17
2020 : 340 : : $101 : B's share = $17.
2 48
2020 : 960 : : $101 : C's share = $48.

Shorten the work by cancellation.

Proof: $36 + $17 + $48 = $101.

(7) See Key.

$$72 \div 350\tfrac{5}{14} = 72 \div \frac{4905}{14} = \frac{72 \times 14}{4905};\ \text{cancel.}$$

$$62\tfrac{8}{7} \div 350\tfrac{5}{14} = \frac{440}{7} \times \frac{14}{4905};\ \text{cancel.}$$

$$71\tfrac{1}{2} \div 350\tfrac{5}{14} = \frac{143}{2} \times \frac{14}{4905};\ \text{cancel.}$$

$$144 \div 350\tfrac{5}{14} = \frac{144 \times 14}{4905};\ \text{cancel.}$$

(8) (1500 + 2700) : 1500 : : $420 : A's share.
(1500 + 2700) : 2700 : : $420 : B's share.

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(9) $(2000 + 2400 + 2400) : 2000 :: \$340 :$ A's share.
$(2000 + 2400 + 2400) : 2400 :: \$340 :$ B's share.
$(2000 + 2400 + 2400) : 2400 :: \$340 :$ C's share.

407. (1) Many teachers prefer to solve all problems of this kind by the method given in Arts. 379 and 380 of the Arithmetic.

$$20 \text{ yd. } 18 \text{ in. } 4 \text{ ft. } 3 \text{ da. } 10 \text{ hr. } \overset{\text{men}}{\underline{27}}$$

The work is arranged as shown above, 72 men being written last, as the number of men is required. The effect of making the length of the ditch 1 yd. is first considered, 1 yd. being written under 20 yd. As this would require $\frac{1}{20}$ of the number of men, 20 is written under 72 men as a divisor. To dig a ditch 1 in. wide would require $\frac{1}{18}$ of the number, and to dig it 1 ft. deep would require $\frac{1}{4}$ of the number; 18 and 4 are written below the line as divisors. To do it in 1 da. would require 3 times as many men; 3, therefore, is written above the line as a multiplier. Working 1 hr. a day would require 10 times as many men; 10, therefore, is written as a multiplier. At this stage, the work appears as follows:

$$20 \text{ yd. } 18 \text{ in. } 4 \text{ ft. } 3 \text{ da. } 10 \text{ hr. } \overset{\text{men}}{\underline{72 \times 3 \times 10}}$$
$$1 \text{ yd. } 1 \text{ in. } 1 \text{ ft. } 1 \text{ da. } 1 \text{ hr. } \underline{20 \times 18 \times 4},$$

the product of $72 \times 3 \times 10$ divided by the product of $20 \times 18 \times 4$ indicating the number of men needed to dig a ditch 1 yd. long, 1 in. wide, 1 ft. deep, in 1 da., working 1 hr. per day.

The next step is to consider the effect of making the length of the ditch, 30 yd. instead of 1 yd., the former number being written in the third line under 1 yd. This would require 30 times as many men; 30, therefore, is placed above the line. To dig a ditch 27 in. wide would require 27 times as many, and 5 ft. deep would require 5 times as many; 27 and 5, therefore, are written above the line as multipliers. To dig it in 15 da. would require $\frac{1}{15}$ of the number, and to

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work 9 hr. a day would require $\frac{1}{3}$ of the number; 15 and 9, therefore, are written as divisors. The work now appears in this form:

20 yd. 18 in. 4 ft. 3 da. 10 hr. men
1 yd. 1 in. 1 ft. 1 da. 1 hr. $\frac{72 \times 3 \times 10 \times 30 \times 27 \times 5}{20 \times 18 \times 4 \times 15 \times 9}$,
30 yd. 27 in. 5 ft. 15 da. 9 hr.

from which the result is computed after canceling.

By the proportion method the effect of making the length 30 yd. instead of 20 yd. is first considered and indicated, then the effect of making the width 27 in. instead of 18 in., etc.; by the method of "analysis" the units, 1 yd., 1 in., 1 ft., 1 da., 1 hr., are successively introduced and their effect considered; then the length, width, depth, of the required ditch, with the allowed number of days and the hours per day.

408. (1) Tons: 21 fifths $\div$ 3 tenths = 42 tenths $\div$ 3 tenths.

(2) Number of pairs: $60s. \div 6s.$

(3) If 2 thirds = 8 yr. 4 mo.
1 third = 4 yr. 2 mo.
Add.

(4) 5 qt. = $\frac{5}{8}$ pk. = .625 pk. (5) $\frac{7}{20}$ of \$7.

(6) $\frac{8}{3}$ of $\$\frac{3}{8}$; cancel.

(7) In a year \$50 will produce \$3 interest; to produce \$18 requires 6 yr.

(8) Loss: $\frac{1}{3}$ cost, or $33\frac{1}{3}\%$.

(9) If \$9 is 25% of the cost, 50% of cost is \$18.

(10) 8 qt. $\times$ 7. (11) $\$\frac{2}{3} \times 120$, or $\frac{2}{3}$ of \$120.

(12) Point off three places: \$66.321.

(13) $1\% = \$120 \div \frac{4}{5} = \$120 \times \frac{5}{4} = \$150$; $100\% = \$15000$.

(14) If 8% is gained in a year, to gain 100% will require $\frac{100}{8}$ yr., or $12\frac{1}{2}$ yr. The given principal, \$50, need not be considered.

(15) The insurance for \$100 costs $\$\frac{1}{5}$. Rate, $\frac{1}{5}\%$.

(16) Number of lots: 24 thirds $\div$ 2 thirds. *Ans.* 12 lots.

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(17) $\$75 - \$72$.

(18) He still owns $\frac{1}{8}$ of his former share of $\frac{2}{3}$, or $\frac{2}{3}$.

(19) $\frac{1}{4}$ lb. Troy $= 3$ oz. $= 60$ pwt. Number of rings: 60 pwt. $\div 2\frac{1}{2}$ pwt. $= 120$ half pwt. $\div 5$ half pwt. $= 24$.

Ans. 24 rings.

(20) Number of square inches: $[(10 \times 4) + (10 \times 3) + (4 \times 3)] \times 2 = (40 + 30 + 12) \times 2 = 82 \times 2 = 164$.

409. (7) For answers including days of grace, see the end of the chapter.

415. These exercises should not be dropped after they are finished; they should be taken up occasionally as opportunity offers.

422. The teacher may find it difficult to make some pupils fully comprehend the principles underlying the method of extracting square root. Such pupils should be shown how to perform the operation.

424. The form as given in the Arithmetic is the abbreviated one. The work in full is given herewith.

```
                  6 5 2
               42'51'04
                  36
          125      6 51
                   6 25
         1302        26 04
                     26 04
```

426. (7) 6.'40. (8) .12'10. (9) .14'40. (10) .19'60. (11) .22'50.

431. (3) $B^2 = 39^2 - 15^2$. (4) $H^2 = 20^2 + 21^2$.

433. See Suggestion in Art. 422, above.

434. (9) Extract the cube root of each term of the fraction separately.

(13) Change mixed numbers to improper fractions.

438. (1) The distance XY should measure very nearly 11 in. (2) 7 to 22.

439. Many teachers prefer to use $3\frac{1}{7}$ as the ratio. Using this ratio, the circumference of a circle having a diameter of

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$3\frac{1}{2}$ in. is found to be $3\frac{1}{2}$ in. $\times$ $3\frac{1}{7}$, or $\frac{7}{2}$ in. $\times$ $\frac{22}{7} = 11$ in., which differs from the other answer by less than $\frac{1}{200}$ in.

440. (2) AB measures $1\frac{3}{4}$ in. One-half of the diameter.

(3) One-half of the circumference. (4) $5\frac{1}{2}$ in., $1\frac{3}{4}$ in.

(5) The following rule may be employed: *To find the area of the circle, multiply the square of the radius by* $3\frac{1}{7}$.

441. "1 sq. ft." is written in the first part of the solution in order to obtain 1296 sq. ft. in the multiplicand in the second part, and also in the result. This is considered by many teachers of arithmetic as more systematic than to omit the denomination until the result is reached, and some examiners are inclined to rate as incorrect an answer paper in arithmetic which does not contain the denomination all through the work. It may be advisable to require a certain accuracy of statement throughout the work of the upper grades, provided, however, that it does not impede too much the progress of the pupils. It must be borne in mind, though, that high school pupils are expected to omit the denomination throughout their algebraic work, supplying it only in the result. The rule for finding areas generally provides that "the number of square units in the area is obtained by multiplying the number of lineal units, etc.," which shows that *numbers* alone are dealt with, the final result being written again with the proper denomination. This is the mode given in the solution to No. 3.

442. (1) This method of computing the area of a triangle is omitted from many text-books, owing to the inability of pupils below the high school to understand the reason for the rule. As, however, many triangular areas are to be calculated in later life in which it is difficult to measure the altitude, pupils should know that there is an arithmetical way to ascertain the area when the lengths of the three sides are given.

(2) After this has been worked by the rule pupils may be informed that the triangle is right-angled, the area of which

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is 1 sq. ft. $\times$ 35 $\times$ $\frac{1}{2}$ of 84. This result being the same as the other will show the correctness of the rule.

No. 3 is a right-angled triangle; so is each of the triangles in No. 4.

No. 5 may be divided into two right-angled triangles by a perpendicular bisecting the base. The hypotenuse of each is 25 yd. and the base of each is 15 yd., from which the perpendicular can be calculated, which will be found to be 20 yd. Using this as the altitude, the area can be computed. Area = 1 sq. yd. $\times$ 30 $\times$ $\frac{1}{2}$ of 20. The area should also be obtained by the rule given in No. 1. The method given for No. 5 is suggested in the Arithmetic for No. 8. See Key.

In No. 7 the altitude can be determined as in Nos. 5 and 6: $\sqrt{6^2 - 3^2} = \sqrt{36 - 9} = \sqrt{27} = 5.196+$. Area = 1 sq. ft. $\times$ 5.196 $\times$ $\frac{1}{2}$ of 6 = 15.588 sq. ft. Working by the rule given in No. 1 makes the extraction of the square root the last operation, which is an advantage when the result contains decimals. It is a good practice to require the pupils to draw to scale a triangle such as No. 7, and to compare the measured length of the altitude with the calculated one. If each side be made 6 in. long, the altitude should measure about $5\frac{3}{16}$ in. (5.1875 in.).

443. See Key.

449. (4) The pupil who draws the "development" and folds it into the required prism will have no difficulty in understanding the rule.

452. (2) The pupils should be left to themselves to ascertain the "development," which is made up of four triangles, each having a base of 4 in. and an altitude of 6 in. Some pupils who have not taken up the geometry work of Chapter Eight may cut out of paper four triangles of the given dimensions, which they place side by side and trace the required development. Others may accomplish the same result by cutting out one triangle, using it to trace the four that constitute the development; etc.

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(5, 8) The development need not be drawn by pupils that have not completed the work in Chapter Eight.

(9) Pupils will see that a semicircle can be folded into a cone the slant height of which will be equal to the radius of the semicircle. Since the semicircle is folded so as to constitute the circular base of the cone, the diameter of the base is 3 in., or one-half the diameter of the semicircular sector. If a sector less than a semicircle is taken, the slant height will still equal the radius of the sector, but the diameter of the base will grow smaller as the size of the arc diminishes.

453. The hollow square pyramid should first be made. Its altitude may then be measured, and the prism constructed. If the prism is made first, the pupils may find it difficult to determine the correct slant height of the required pyramid.

(2) 1 cu. in. $\times 10^2 \times \frac{1}{3}$ of 12.

(3) The area of the base is found by the rule given in Art. 442, if the pupils do not recognize it as a right-angled triangle.

(4) 1 cu. yd. $\times 9 \times 13 \times 19 \div 27$.

(7) 1 cu. meter $\times 3^2 \times 3.1416 \times \frac{1}{3}$ of 18.

461. (15) 1 qt. $= \frac{1}{4}$ of 231 cu. in., or $57\frac{3}{4}$ cu. in.

462. (17) Cancel.

466. (2) $\$\frac{1}{2} \times 100 \times 50$.

(7) 12 in. $\times (1\frac{1}{2} + 1 + 1\frac{1}{2} + 1)$. (8) $\$\frac{1}{2} \times 91$.

467. (4) £161 17*s.* 6*d.* $\div$ £1 1*s.* 7*d.* $= 38850d. \div 259d. = 150$. *Ans.* 150 bbl.

(13) 13 ft. $\times$ 527. Reduce to miles, etc.

(20) M does $\frac{1}{4}$ of the work in a day, N $\frac{1}{5}$, and O $\frac{1}{6}$. Together they do in a day $\frac{1}{4} + \frac{1}{5} + \frac{1}{6}$, or $\frac{37}{60}$ of the work. To do 60 sixtieths of the work will require as many days as 60 sixtieths contains 37 sixtieths, or $1\frac{23}{37}$ da.

469. (1) Number of shares: $\$50{,}000 \div \$100$, or 500. The owner of 10 shares owns $\frac{1}{50}$ of the stock.

(2) $\frac{1}{50}$ of $\$2000$.

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(3) $2000 = 4\%$ of the capital, $50,000.

471. The rate in these examples is the per cent, the par value being multiplied by it to obtain the price per share.

(2) $\frac{1}{8}\%$ of $27,500 = $27,500 \div 800 = $275 \div 8$.

473. (1) $4 \div .95 = 400 \div 95$; cancel.

(10) He receives $5 interest per year, or $30 in six years, and $100 for the bond; a total of $130. Deducting the cost of the bond, $104, leaves $26 as the sum really obtained for the interest for 6 years.

484. (2) See No. 1, Key.

486. Calculations in the countries using the metric system are much simpler in certain lines. The work in denominate numbers, which in this country employs so many tables, is done in the metric system by "pointing off." There are in the latter no separate tables of avoirdupois and Troy weights, or of dry and liquid measures. The relations, too, between the unit of measure (the meter), that of capacity (the liter), and that of weight (the gram) are decimal. A cube 1 meter each way contains 1000 liters; and a liter of water weighs 1000 grams. In the United States, the gallon contains 231 cu. in., and a gallon of water weighs about $8\frac{1}{3}$ lb. A bushel contains 2150.42 cu. in. The weight of a cubic foot of water, 1000 ounces, is about the only decimal feature of our tables.

489. These problems present nothing difficult save the names of the denominations, which are unfamiliar.

497. See Key also. (1) Changing the fractions to a common denominator, the problem becomes: If 10 fifteenths cost $4.50, what will 12 fifteenths cost? The operation is $\frac{12}{10}$ of $4.50, or $\frac{6}{5}$ of $4.50.

(2) $\frac{1}{8}$ of 4 da., or $\frac{1}{2}$ da.

(3) Divide $12 by 30¢, the interest on $1 for 5 years.

Ans. $40.

(5) $(15 + 30 + 5)$ da.

(7) $(\$\frac{1}{2} \times 460) - (2¢ \times 460) = \$230 - \$9.20$.

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(8) There are 32 gi. in a gallon. *Ans.* $\frac{3}{4}$ lb. $\times$ 32.

(9) 15 lb. $\times$ 144. Use 15 as the multiplier.

(11) 960 lb. = 16 bu. *Ans.* 16 times \$1.05.

(12) At 1%, \$400 will produce \$4 per year, and \$5 in in $1\frac{1}{4}$ yr. To produce \$37.50 the rate must be as many times 1% as \$37.50 is times \$5. *Ans.* $7\frac{1}{2}$%.

(13) $\$10400 \times .01\frac{3}{8} = \$104 \times 1\frac{3}{8} = \$104 \times \frac{11}{8} = \$13 \times 11 = \$143$, *Ans.*

(14) $\$25 \times 3.28 = \$\frac{1}{4} \times 328$, or \$82.

(15) B's gain is 7 twelfths or \$1400. A's gain is 5 twelfths, which is $\frac{5}{7}$ of B's gain. $\frac{5}{7}$ of \$1400 = \$1000. It is not necessary to find the whole gain.

(16) $2° 3' \times 15$.

(17) 20 ft. long, 12 ft. wide. Area, 240 sq. ft.

(18) $\frac{5}{9} = \frac{1}{9}$ of 5; therefore, $\frac{5}{9}$ yd. will cost $\frac{1}{9}$ of 90¢. Do not find cost of 1 yd.

(19) $\frac{1}{400}$ of amount of insurance = \$15. Insurance = $\$15 \times 400$, or \$6000.

(20) Loss 9 oz., or $\frac{1}{4}$ of 36 oz. *Ans.* 75%.

498. (7) $\frac{\text{rods } 24 \times 5 \times 8}{4 \times 20}$.

499. (1) ($\frac{1}{4}$ of 8400) − 84. In solving, the pupil should say 21 hundred less 84, not 2 thousand 1 hundred. (3) 900 − 36. (4) 2400 − 48. (5) 4200 − 84. (6) ($\frac{3}{4}$ of 84 hundred) − 84. (7) 3600 − 48. (9) 1200 − 24. (10) 900 − 18. (11) 1200 − ($\frac{1}{4}$ of 48). (12) 900 − 9. (13) 1200 − 6. (14) 900 − 12. (16) $48 \times (12\frac{1}{2} - 1) = 600 - 48$. (17) ($\frac{7}{8}$ of 4800) − 48 = 4200 − 48. (18) ($\frac{3}{8}$ of 4800) − ($\frac{1}{4}$ of 48) = 1800 − 12.

(2) In dividing by $\frac{3}{4}$, etc., many business men multiply by $1\frac{1}{3}$, rather than by $\frac{4}{3}$, especially when the multiplicand is not a multiple of 3. Thus, $37 \div \frac{3}{4} = 37 + 12\frac{1}{3}$, or $49\frac{1}{3}$; that is they add to 37 one-third of 37. (3) $\frac{4}{5}$ of 36. (5) $\frac{3}{4}$ of 36. (6) $\frac{2}{3}$ of 36. (7–12) See Arithmetic, Art. 296

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(13–18) Change to improper fractions and reject the denominators: $49 \div 7$, $33 \div 3$, $40 \div 10$, $72 \div 6$, $63 \div 9$, $35 \div 7$.

500. The importance of accuracy has been discussed in Art. 309 of the Manual, p. 64, and the advisability of performing an operation a second time in a different way. Some of the short methods afford a second way of obtaining a product, which may be used to prove the correctness of the product first obtained.

(7) $\frac{1}{8}$ of 848000.

(8) $375000 - 375$

(10) 457×16

2742

7312, *Ans.*

(12) 983×51

4915

50133, *Ans.*

Do not place the multiplier under the multiplicand; use the latter as the product by 1 without rewriting it.

(14) $176 \times 23\frac{3}{4}$

	528	product by 3
Divide by 4	132	product by $\frac{3}{4}$
	352	product by 2 tens
	4180,	*Ans.*

(16) $895 \times 44\frac{4}{5}$

	3580	product by 4
Divide by 5	716	product by $\frac{4}{5}$
	3580	product by 4 tens
	40096,	*Ans.*

When the numerator of the fraction is the same as the unit's figure of the multiplier, one line is saved by doing the work in the way given above.

501. (6) Divide $\frac{3}{5}$ by 160. 2000 lb. $\times \frac{7}{64} = 218\frac{3}{4}$ lb. = 218 lb. 12 oz.

(11) $(2\frac{2}{13} \times 16\frac{5}{7}) \div (1\frac{1}{2}$ of $2\frac{2}{5}) = (\frac{28}{13} \times \frac{117}{7}) \div (\frac{3}{2} \times \frac{12}{5}) = \frac{28}{13} \times \frac{117}{7} \times \frac{2}{3} \times \frac{5}{12}$. In all examples limited to multiplica-

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tion and division, the entire work should first be indicated. Care must be taken to invert the whole divisor.

502–504. Many of these exercises involve the change of the $\frac{1}{2}$ yd. to feet and inches to avoid a fraction in the result. They are interesting rather than useful or practical.

511. (16) $270 \times \frac{x}{100} \times \frac{109}{360} = 273.27 - 270 = 3.27.$

$$x = \frac{3.27 \times 100 \times 360}{270 \times 109} = 4. \quad \textit{Ans.}\ 4\%.$$

(17) $360 \times \frac{9}{200} \times x = 25.11.$

$$x = \frac{25.11 \times 200}{360 \times 9} = \frac{31}{20}.$$

Ans. $\frac{31}{20}$ yr. = 1 yr. 6 mo. 18 da.

512. (4) $600 \times \frac{x}{100} \times \frac{21}{360} = 2.45.$

$$x = \frac{2.45 \times 100 \times 360}{600 \times 21} = \frac{245 \times 360}{600 \times 21}.$$

The decimal point in the numerator (2.45) may be dropped by dropping the multiplier 100.

(6) $x \times \frac{6}{100} \times \frac{132}{360} = \frac{11x}{500}$, the discount.

$x - \frac{11x}{500} = \frac{489x}{500}$, the proceeds.

$$\frac{489x}{500} = 2689.50.$$

$$x = \frac{268950 \times 5}{489} = 2750. \quad \textit{Ans.}\ \$2750.$$

(7) Discount, \$150 − \$147.75 = \$2.25

$$150 \times \frac{6}{100} \times x = 2.25$$

$$x = \frac{2.25 \times 100}{150 \times 6} = \frac{1}{4}. \quad \textit{Ans.}\ \frac{1}{4} \text{ yr., or } 90 \text{ da.}$$

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(8) Let x represent the term in days:

$$1650 \times \frac{6}{100} \times \frac{x}{360} = 4.95.$$

$$x = \frac{4.95 \times 100 \times 360}{1650 \times 6} = \frac{495 \times 360}{1650 \times 6} = 18$$

Ans. 18 da.

(9)
$$x \times \frac{6}{100} \times \frac{69}{360} = \text{discount} = \frac{23x}{2000}$$

$$x - \frac{23x}{2000} = \text{proceeds} = \frac{1977x}{2000} = 469.30$$

$$x = \frac{469.30 \times 2000}{1977} = \frac{938600}{1977} = 474.76-$$

Ans. \$474.76-

The answers to these problems should be proved in every case. In No. 1, for instance, the pupil should calculate the discount, using the term just found; if the result is \$ 6.30, the answer is correct. Use the face found in No. 2 and calculate the proceeds, which should be \$ 397.80. In No. 3, find the discount (and proceeds) by the aliquot part method, and also by the 6% method; if the results correspond, the answer is correct.

514. (5) 16 + (3 times 16) + (5 times 16) = 9 times 16. (6) The pupil can use dimensions one-fourth of those given, say 3 ft. and 4 ft. The result, 5 ft., is then multiplied by 4. (7) \$ 35 + $\frac{1}{8}$ of \$ 35 = \$ 35 + \$ 11$\frac{2}{3}$. (19) (5280 ÷ 16) revolutions. (68) $6\frac{1}{4}¢ \times 10000 = \$\, 6\frac{1}{4} \times 100$. (87) $\frac{1}{2}$ of 8 bu. (108) The profits are one-half the capital. A's share is $\frac{1}{2}$ of \$ 600; B's, $\frac{1}{2}$ of \$ 400. (109) 12 weeks at 20¢ per week. (112) $(\$\, \frac{1}{2} \times 84) - 84¢ = \$\, 42 - 84¢$. (122) Move the decimal point three places to the right.

515. (4) Cost = 90% of value; selling price = 110% of value; gain = 20% of value, which is $\frac{20}{90}$, or $\frac{2}{9}$ of cost. *Ans.* $22\frac{2}{9}$%.

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(42) $\frac{1}{3} \div .00003\frac{1}{3} = \frac{1}{3} \div \frac{3\frac{1}{3}}{100000} = \frac{1}{3} \div \frac{10}{300000} = \frac{1}{3} \times \frac{300000}{10}$
$= 10000$, *Ans.*

(45) $\frac{3}{4}$ of $17\frac{5}{8} \div \frac{4}{5}$ of $\frac{35}{1000} = \frac{3}{4} \times \frac{141}{8} \times \frac{5}{4} \times \frac{1000}{35}$; cancel.

(88) 1411734 = number of pounds

4/8665)6870205/0000
200370
57105
84400
357350
166950
209550
14890 remainder
20
48665)297800(6 = number of shillings
5810 remainder
12
48665)69720(1+ = number of pence

Ans. £ 1411734 6*s.* 1*d.*

The first remainder indicates £ $\frac{14890}{48665}$, which is reduced to shillings by multiplying by 20; the product of 20 times the numerator is first obtained (without writing out the fraction) and 48,665 is continued as the divisor. The reduction to pence is made in the same way.

(114) 400009.000786 — 4009.0786 — 4009.000786.

The average pupil would receive more benefit from the algebraic and construction work of Chapters Seven and Eight, respectively, than is obtainable from the study of compound proportion, cube root, longitude and solar time, exchange, annual interest, and stocks and bonds in this chapter.

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APPENDIX TO CHAPTER SIX.

Days of Grace.

See Appendix, Chapter Five, page 75.

409. (7) Note is due Aug. 16. Term of discount from May 23 to Aug. 16, 85 da. Discount, $12.56+. Proceeds, $747.44−.

467. (19) Due July 22. Term of discount, 48 da. Discount, $3. Proceeds, $372.

498. (8) Representing the face of the note by x, the discount for 63 da. will be $x \times \frac{63}{360} \times \frac{6}{100}$, or $\frac{21x}{2000}$. The proceeds will be $x - \frac{21x}{2000}$, or $\frac{1979x}{2000}$.

$$\frac{1979x}{2000} = 294.75$$
$$1979x = 589500$$
$$x = 297.88+$$
Ans. $297.88+

Or, the discount on $1 for 63 da. at 6% is $.0105, and the proceeds are $.9895. To obtain proceeds of $294.75 the face of the note must be for as many dollars as 294.75 contains .9895; etc.

(10) Due Aug. 6, 1904. Terms of discount, July 1 to Aug. 6, 36 da. Discount, $3. Proceeds, $497.

514. (73) Page 399. Interest on $400 for 60 da. is $4; for 3 da. it is $\frac{1}{20}$ of $4, or 20¢; for 63 da., it is $4.20, *Ans.*

(90) The term of a 65 da. note with grace is 68 da. Interest for 60 da., $10; for 6 da., $1; for 2 da., 33⅓¢. *Ans.* $11.33⅓.

515. (3) Discount for 93 da.: $x \times \frac{6}{100} \times \frac{93}{360} = \frac{31x}{2000}$. Proceeds: $x - \frac{31x}{2000} = \frac{1969x}{2000}$.

$$\frac{1969x}{2000} = 240$$
$$1969x = 48000$$
$$x = 243.78-$$
Ans. $243.78−

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(20) Page 404. Selling price of cloth: $\$2\frac{1}{4} \times 672$, or \$1512. Discount for 63 da. $\frac{\$1512 \times 7 \times 63}{100 \times 360} = \$20.27+$.

Proceeds: $\$1512 - \$20.27+ = \$1491.73-$.
Buying price of cloth: $\$1\frac{1}{4} \times 672 = \$840$.
Profit: $\$1491.73 - \$840 = \$651.73$.

(126) Page 413. Due Aug. 16. Interest from Feb. 13 to Aug. 16, 184 da. at 6%, \$32.20. Amount due Aug. 16, \$1050 + \$32.20 = \$1082.20. Term of discount, May 13 to Aug. 16, 95 da. Interest (discount) on \$1082.20 for 95 da. at 8%, \$22.85-. Proceeds, \$1082.20 - \$22.85- = \$1059.35+, *Ans.*

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CHAPTER SEVEN.

WALSH'S NEW GRAMMAR SCHOOL ARITHMETIC, PAGES 415–479.

This treatment of algebraic equations is so elementary that no teacher need hesitate because of any supposed difficulty in the subject-matter. Arts. 516–549 are recommended for study at the beginning of the seventh year, as a preliminary to the "problems" in percentage and interest in Chapter Five. The remainder of this chapter may be taken up in the eighth year in place of some topics of Chapter Six that are recommended for omission.

516. See Manual, Art. 309, and Key.

517. (1) $x+5=7$.
(2) $6+x=9$.
(3) $x-4=1$.
(4) $12-x=8$.
(5) $10-x=3$.
(6) $x-10=3$.
(7) $x+3=15$.
(8) $x+2=11$.
(9) $x-5=8$.
(10) $x+5=15$.
(11) $x-10=15$.
(12) $10-x=7$.
(13) $x+15=40$.
(14) $x+20=60$.
(15) $x+2000=3200$.

Some pupils may give different answers to some of the foregoing without being altogether incorrect. No. 8, for instance, the pupil may write $x=11-2$; or No. 11, $x=15+10$. Strictly speaking, these do not so accurately indicate the conditions expressed in the exercises, but such equations should be accepted for the present.

NOTE. — The denomination should not be written in any case.

518. See Key. It is not expected, as yet, that the pupils will solve these by transposing; this will come later. See Art. 532.

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520. See Answer Book.

521. (3) Becomes $5x=40$; (6) $18x=36$; (7) $x=72$; (8) $9x=27$; (9) $6x=54$; etc.

522. (1) 5. (2) 1. (3) 12. (4) 8. (5) 2. (6) 6. (7) 13. (8) 11. (9) 42. (10) $a+c$. (11) bc. (12) $bc+a$. (13) $bc-a$. (14) $c-b$.

523. The pupils should be required at first to write out the work as fully as is shown in the sample solution to the first problem. While pupils are supposed, as a rule, to omit all unnecessary figures, it is advisable at the commencement of this new subject to indicate the successive steps, even though the problem can be solved without a pencil. The resulting equations given in the Key should, in the pupils' papers, be preceded by the necessary preliminary statements.

525. See Answer Book.

526. These exercises are to be solved arithmetically to lead up to the algebraic expressions on the next page.

$$(10)\ \frac{1}{3}x+\frac{1}{4}x=\frac{7}{12}x. \qquad \frac{x}{3}+\frac{x}{4}=\frac{7x}{12}.$$

528. See Key. In working Nos. 5–16 at sight, fractions in the first member should be combined before the equation is cleared of fractions.

(5) $\frac{3w}{4}=12$. (6) $\frac{5x}{6}=5$. (7) $\frac{y}{2}=10$. Etc.

529. In the written work, it is usual to commence by clearing the equation of fractions. This should be done by the pupils, although it is not the shortest way in the present exercises.

(3) $3x+2x=210$.
(4) $4x+3x=588$.
(5) $3x+4x=168$.
(6) $21x+25x=3220$.
(7) $8x+9x=1224$.
(8) $23x=920$.

Etc.

530. Some pupils may wonder at being required to employ a long method to solve No. 2, which they can work

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arithmetically without a pencil. Algebra is used to solve problems that cannot be worked by the ordinary arithmetical methods, but the beginnings of algebra should connect the work already had with the new work. It will also help the slower pupils to understand the arithmetical solutions that they have employed without being fully able to grasp the reasons for the methods used.

Clearing of fractions: (1) $2x+3x=200$. (2) $8x-x=5712$. (3) $3x+x=720$. (9) The detailed form of solution given in the Key should be required from the pupils in each problem.

533. The pupils should be required, before finding the result, to rewrite the given equation with all the unknown quantities in the first member and the known quantities in the second; they should then combine the quantities; lastly, they should find the value of the unknown quantity. It is understood that the first step should be to clear the equation of fractions, if there are any.

(2)	$4x-5=83.$	(6) $5x=x+40.$
Transposing:	$4x=83+5.$	$5x-x=40.$
Combining:	$4x=88.$	$4x=40.$
Dividing:	$x=22.$	$x=10.$

(9)
$$3x-6=48+x.$$
$$3x-x=48+6.$$
$$2x=54.$$
$$x=27.$$

(17)
$$2x-11+6x-60=5x+25.$$
$$2x+6x-5x=25+11+60.$$
$$3x=96.$$
$$x=32.$$

(19) $2x-6=16+\frac{x}{2}-\frac{x}{3}$. (20) $2x+\frac{3x}{5}-\frac{x}{2}=\frac{3x}{4}+27$.

Clearing of fractions:

$12x-36=96+3x-2x$. $40x+12x-10x=15x+540$.

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Transposing:

$12x-3x+2x=96+36.$ $40x+12x-10x-15x=540.$

Combining: $11x=132.$ $27x=540.$

Dividing: $x=12.$ $x=20.$

534. (2) Some pupils may write the equation in this form: $45+4x=7x$, which would give them $4x-7x=-45$ after transposing. As they have not yet learned to combine $4x$ and $-7x$, it is advisable that they should write the equation in the form given in the text-book: $7x=45+4x$.

(18) The second method given in the Key should be followed: $x=$ less; $3x+11=$ greater.

$3x+11-x=33.$

538. (2) This problem asks, "What was his *gain?*" As there was a *loss* of $25, this may be expressed as a *gain* by prefixing the minus sign; thus, $-$25 gain.

(4) He saved $-$10.

(6) He was -4 mi. north of his starting point.

(7) A loss of $5. (8) 5 mi. west.

(9) The woman is 3 in. shorter than her husband.

(10) Mary is -3 yr. older than John.

542. (2) See diagram, p. 362, Arithmetic, Prob. 11.

(3) $200. (4) $1000.

548. See Key for the equations as they will stand after clearing of fractions.

550. These are to be worked at sight by the pupils in their own way.

552. (2) The cost of the oranges is obtained in the first equation, $3x+8y=25$, by substituting for $8y$ its value, 16; thus, $3x+16=25$, $3x=25-16$, $3x=9$, $x=3$. The oranges cost 3¢ each. To prove the correctness of the answer, substitute these values in the second equation, $3x+4y=17$; thus, $9+8=17$.

553. Prove the correctness of the answers by substituting both values in the equation not used to obtain the value of x.

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Thus, the value of y is found to be 8; substituting the value of y in $x+y=15$, we have $x+8=15$, $x=7$. Substitute both values in the other equation, $2x+3y=38$; thus, $14+24=38$, which shows the results are correct. It would not prove the accuracy of the work to substitute both values in $x+y=15$, as the value of x was obtained from this. If, for instance, the value of y was incorrectly found to be 9, the first equation would give the value of x as 6; and the use of the same equation would only repeat $6+9=15$. The substitution of these values in the second equation would show that either is incorrect, since $12+27$ is not 38.

555. (6) See Key. $2x+y=42$; $2x+4y=96$.

(8) Clearing of fractions (a): $26+5x-6y=52y-39x$.
Transposing (a): $5x-6y-52y+39x=-26$.
Combining (a): $44x-58y=-26$.
Clearing of fractions (b): $144+10x-12y=24y+9x-6y$.
Transposing (b): $10x-12y-24y-9x+6y=-144$.
Combining (b): $x-30y=-144$.
The equations now are $44x-58y=-26$ and $x-30y=-144$.

558. (1) 4. (2) 6. (3) 12. (4) 13. (5) 1. (6) 25. (7) 14. (8) 22. (9) 68. (10) 3. (11) 3. (12) 2.

560. (1) 20. (2) 100. (3) 18. (4) 11. (5) 192. (7) 126. (8) 84. (9) 150. (10) 180. (11) 7. (12) 4.

562. (1) $15a^2$. (2) $6a^2b$. (3) $6ab^2$. (4) $6a^3b$. (5) $6a^2b^2$. (6) $6a^2b^3$. (7) $15a^2bc$. (8) $15a^2bc$. (9) $15a^2b^3c^3$. (10) $a^2b^3c^4$. (11) $3a^3b$. (12) $5a^3b$.

563. (5) $-15a^2$. (6) $8a^2$. (7) $8a^2$. (8) $-6a^2b^2$. (9) $15a^2bc$. (10) $-15a^2bc$. (11) $-15a^2bc$. (12) $5a^2b$.

565. (1) The $3x$ of the second partial product is placed under the $2x$ of the first partial product. See No. 2.

568. (5) 1. (6) 2. (7) 3. (8) 2. (9) 3. (10) 4.

570. 2, 5, 7, and 10 contain like terms.

574. In applying the rule to write the letters of the dividend, it is understood that any letter is omitted if the divisor

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contains the same letter with the same exponent. The coefficient 1 need not be written in the result. See No. 4, the answer to which would be $1\,a^0x$, following the rule literally; but a^0 being equal to 1, the given result, x, is the same as $1 \times 1 \times x$.

575. This rule is the same as that given for multiplication, Art. 563. It is inadvisable to attempt to make the pupils understand the underlying principles.

597. Each of the expressions should first be examined for monomial factors, as in Nos. 1, 2, 3, 4, 7, 8, 9, 10, etc. (see Answers). The other factor or the original quantity, if a binomial, may be a difference of squares, as in Nos. 13, 16, 20, 35, 36, and 47 (see Art. 586); the sum or the difference of cubes, as in Nos. 5, 7, 9, 18, 30, 31, and 48 (see Art. 594); or it may have no factors, as in Nos. 1, 8, 12, 19, 23, 27, 34, 37, and 49. In factoring No. 46, we first obtain $(m^2+4)(m^2-4)$, the latter of which is a difference of squares, equal to $(m+2)(m-2)$. If the original expression or one factor is a trinomial, it should be examined to ascertain if it is the square of the sum, as in Nos. 4, 6, 11, 24, etc. (see Art. 582); the square of the difference, as in Nos. 2, 15, etc. (see Art. 584); or the product of two unequal binomials as in Nos. 3, 10, 14, 17, 21, 22, 38, 43, and 44 (see Art. 588). No. 25 may be considered a trinomial; it is the square of the sum of $x+y$ and 1; No. 26 is a similar square of the difference. No. 28 is in the form of a binomial difference of the squares of $x+y$ and 1, the factors being $(x+y)+1$ and $(x+y)-1$, which are rewritten $(x+y+1)(x+y-1)$; Nos. 29, 40, and 41 may be written in the same form. No. 32 is first factored into $(x^2-4)(x^2-1)$, each of which is a difference of squares, which may be again factored. No. 33 is factored into $(a^2+2)(a^2-4)$, the second of which is a difference of squares. No. 39 becomes $a(x^2-1+2\,bx+b^2)$, the polynomial factor of which may be written $x^2+2\,bx+b^2-1$, or $(x+b)^2-1$, which is a

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difference of squares equal to $[(x+b)+1][(x+b)-1]$, or $(x+b+1)(x+b-1)$. No. 42 can be written $a^4+2a^2+1-a^2$, which is factored in the same way as No. 41. In No. 45 the monomial factor is z, the other being z^4+z^2+1, which is changed to the form given in No. 41, and again factored. No. 50 becomes $(a^3+8)(a^3-8)$ each factor of which may be again factored, the former being a sum of two cubes and the latter a difference of two cubes.

599. (1) $\frac{2a}{3}$. (2) $\frac{2b}{3a}$. (3) $\frac{b}{3x}$. (4) 7. (5) $\frac{x+y}{3b}$. (6) $2ab^2$. (7) $\frac{2b^2}{1+b}$. (8) $\frac{a+2b}{3b}$. (9) $\frac{ny+1}{a+x}$. (10) $\frac{2p+3y}{p+y}$.

600. (1) $\frac{bx}{3x}$. (2) $\frac{2xy}{3x^2}$. (3) $\frac{2ac}{3c}$. (4) $\frac{10nxy}{14mn}$. (5) $\frac{4a^2}{a}$. (6) $\frac{16a^3}{4a^2}$. (7) $\frac{by+bz}{2ab}$. (8) $\frac{2ab^3}{b}$. (9) $\frac{9m^2t^2}{3mt}$. (10) $\frac{2a^2b+2ab^2}{2ab}$.

603. (10) See Key. $20(x+1)=30(x-1)$; $20x+20=30x-30$; $20x-30x=-30-20$; $-10x=-50$; $x=5$, *Ans.*

(19) $x^2-2x-63=x^2-8x+15$; $-2x+8x=15+63$; $6x=78$; $x=13$, *Ans.*

In Nos. 9, 10, 18, 19, and 20, it is not necessary to find the common denominator. In No. 9, the common denominator is $(z-3)(z-9)$; multiplying the first term by this gives $(z+7)(z-9)$, multiplying the second by it gives $(z-5)(z-9)$. In practice, however, when each term consists of a single fraction, the numerator of the fraction in the first term is multiplied by the denominator of the fraction in the second term, and the numerator of the fraction in the second term is multiplied by the denominator of the fraction in the first term.

604. The negative results in these problems may be neglected.

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607. (1) $x+3$. (2) $x-7$. (3) $x-9$. (4) $x+10$. (5) $a+b$. (6) $x+y$. (7) $x-y$. (8) $a-b$. (9) $x-12$. (10) $x+11$.

612. (4) $x^2+18x+81=115+81=196$.

$x+9=\pm 14$; etc.

(5) $x^2-14x+49=-13+49=36$.

$x-7=\pm 6$; etc.

(6) $x^2=10x$; $x=10$.

(7) Add 100 to both terms. (8) Add 169.

(9) Divide by x; see No. 6. (10) Add 16. (11) Add 4.

(12) Add 225. (13) Add 196. (14) Add 121.

(15) Transposing, $x^2-16x+64=0$.

Extracting the square root, $x-8=0$.

Transposing, $x=8$, *Ans.*

(16) Add 324.

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CHAPTER EIGHT.

WALSH'S NEW GRAMMAR SCHOOL ARITHMETIC, PAGES 480–503.

These exercises are intended to give the pupil an opportunity to obtain by drawing, etc., a knowledge of the more important geometrical facts. He discovers the nature of an angle, the number of degrees in two supplementary angles, the number in a right angle, the number in the sum of four angles formed by two intersecting lines, the equality of opposite angles, the equality of angles formed by parallel lines cut by another line, the equality of the angles and the sides of an equilateral triangle, the equality of the base angles of an isosceles triangle and of the opposite sides, the inequality of the angles of a scalene triangle, etc., etc., etc. He learns to construct the various triangles, quadrilaterals, regular polygons, etc.; to bisect lines, arcs, and angles; to construct angles of certain sizes; to inscribe and circumscribe polygons; etc.

Suggestions as to the method of doing the work will be found on pages 129–150 of the Key.

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